Chhattisgarh Institute of Technology, Jashpur (C.G.)

Department of Electronics & Telecommunication Engg.

Subject- Network Analysis (NA)

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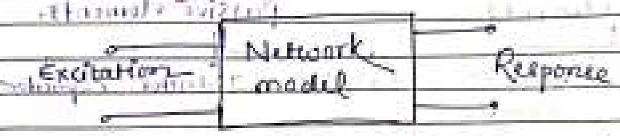
Voltage :- एक प्रकार का बल होता है जो किसी विद्युत परिपथ में इलेक्ट्रान को धक्का लगाने का कार्य करता है।

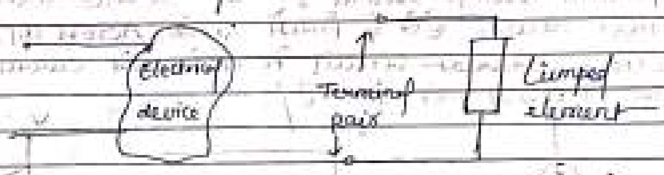
Voltage को 'V' से प्रदर्शित करते हैं।
Unit = Volt.

Network Theory

Network model :-



Network theory के माध्यम से हम किसी network model का response अलग-अलग i/p पर देख सकते हैं।



- **Electric Charge :-** Electrical energy को हम electric charge के रूप में मापते हैं।

$$\boxed{i = \frac{dq}{dt}}$$ q = Charge

Current प्रवाह की प्रमुख तथ्य :-

i) Current के पास direction होता है।
ii) Current का प्रवाह +ve Charge और -ve Charge के द्वारा होता है, और प्रायः Current का direction +ve Charge की प्रवाह की दिशा में होता है।

$$\boxed{V = \frac{dW}{dQ}}$$

Where V = voltage in volts
W = energy in Joule
Q = Charge in Coulombs.

$$\boxed{P = V \cdot i}$$

$$P = \frac{V^2}{R}$$

$$P = I^2 R$$

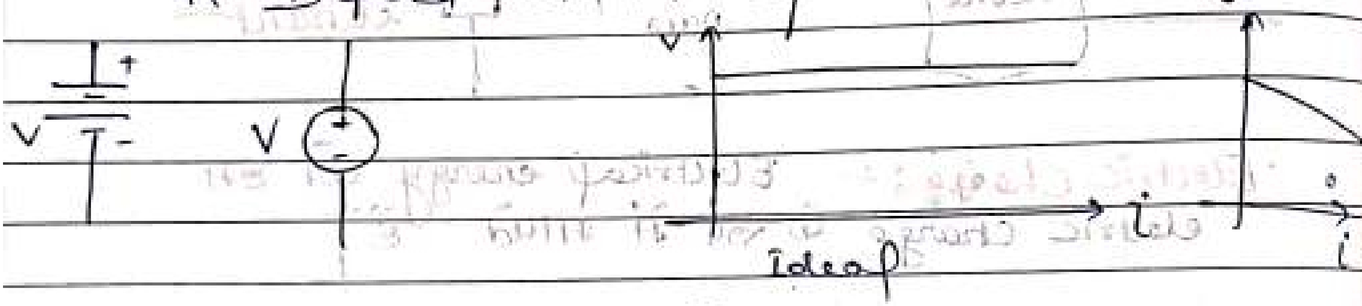
Network elements :-

Active Elements
- Voltage Source
- Current Source

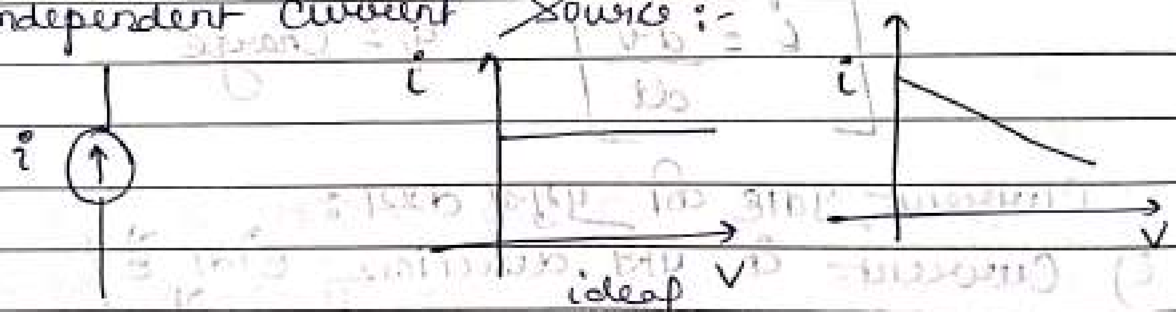
Passive elements.
- Resistor
- Inductor
- Capacitor

• Active elements :- वैसे elements जो कि circuit को लंबे समय तक current या voltage दे पाए Active element कहलाते हैं।

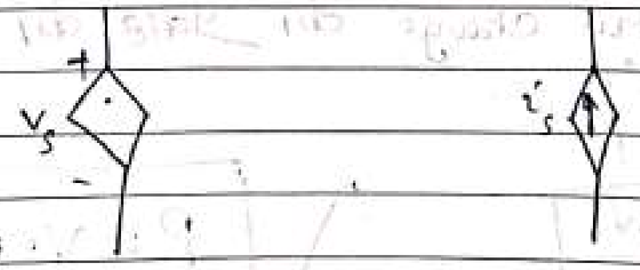
✓ Independent Voltage Source :- ये source वैसे voltage source होते हैं जिसमें इसके Across का voltage ~~current~~ circuit में बहने वाले current पर Depend नहीं करता है।



✓ Independent current source :-



• Dependent ~~current~~ Source/controlled Source :-



• Passive elements :- वैसे elements जो Power को dissipate करते हैं, Passive element कहलाते हैं।

There are 3 Passive elements —

i) Resistor ii) Inductor iii) Capacitor

i) Resistor :—

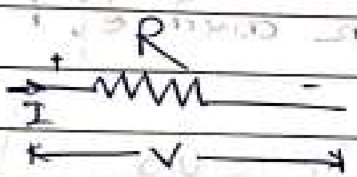
Ohm's Law :— उसके Across

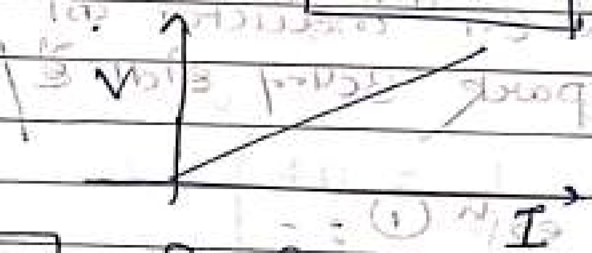
~~"किसी भी चालक में Voltage, उसमें बहने वाले Current के Proportional होता है।"~~

ALT

"Ohm's law के अनुसार यदि ताप और अन्य भौतिक अवस्थायें निश्चित रखीं जाये, तो किसी प्रतिरोधक के सिरों के बीच उत्पन्न विभवान्तर उससे प्रवाहित धारा के समानुपाती होता है।"

$$V \propto I \Rightarrow \boxed{V = RI}$$





- $\boxed{R = \frac{\rho l}{A}}$ ρ = Resistivity (Ω-m).

- G = conductance $= \frac{1}{R} = \frac{i}{V}$ ⇒ Unit = Siemens (S)

ii) Inductor :— यह एक Passive element है, जो magnetic field के form में energy को Store करता है।

Self-inductance :-

$$\boxed{V = L\frac{di}{dt}} \quad (1) \qquad \boxed{L = \frac{\mu N^2 A}{l}}$$

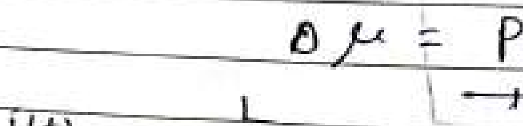
Where,

A = Cross Sectional area of coil (m^2)

N = no. of turns of coil

l = length of coil

μ = Permeability of material inside the coil. for free Space $\mu_0 = 4\pi \times 10^{-7}$ H/m



From eqⁿ ① :-

- यदि current constant होता है, तो voltage inductor के Across zero हो जाता है और inductor short circuit अथवा एक zero resistance wire की तरह behave करने लगता है।
- यदि inductor में instantaneously अथवा suddenly current change होता है, तो rate of change of current ($\frac{di}{dt}$) ∞ हो जाता है, अतः inductor rate of change of current को रोकता है। अतः जब हम inductor को open करते हैं, तो spark उत्पन्न होता है।

from eqⁿ ① :-

$$di = \frac{1}{L} v\,dt$$

$$\int_0^i di = \frac{1}{L}\int_{-\infty}^{t} v\,dt$$

$$i = \frac{1}{L}\int_{-\infty}^{t} v\,dt$$

$$p = v \cdot i = Li\frac{di}{dt}$$

$\therefore$ W_L = total energy stored in inductor

$$= \int_{-\infty}^{t} vi\,dt = \int_{-\infty}^{t} iL\frac{di}{dt}\,dt$$

$$= L\int_{0}^{t} i\,di = \frac{1}{2}Li^2$$

$$\therefore \boxed{W_L = \frac{1}{2}LI^2}$$

iii) Capacitance :— दो या दो से अधिक चालकों को एक विद्युत्रोधी माध्यम द्वारा अलग करके समीप रखा जाए, तो यह व्यवस्था Capacitor कहलाती है।

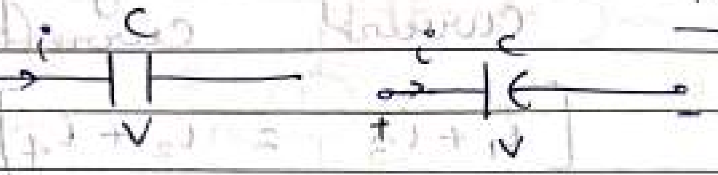
• Capacitor stores energy in the form of electrostatic field.

$$\boxed{C = \frac{\epsilon A}{d}}$$

ϵ = permittivity of insulating material.

ϵ_0 = for vaccum = 8.85×10^{-12} F/m

$$\boxed{i = C\frac{dv}{dt}}$$



$$dv = \frac{1}{C} i\,dt \Rightarrow \int_0^V dv = \frac{1}{C}\int_{-\infty}^{t} \frac{1}{C}\, i\,dt$$

$$\boxed{V = \frac{1}{C}\int_{-\infty}^{t} i\,dt}$$

• यदि capacitor के Across Voltage Constant रहता है, तो current zero होगा।

• यदि voltage अचानक change होता है, current ~~∞ होगा~~ या time = 0 sec इसलिए current = ∞ होगा। इसलिए Capacitor किसी भी voltage के change को रोकता है। इसलिए दो capacitor के charge plate को एक दूसरे से touch करते हैं, तो spark produce होता है।

$$p = v \cdot i = v \cdot C\frac{dv}{dt}$$

W_c = energy stored in Capacitor = $\int_{-\infty}^{t} v i\,dt$

$$= \int_{-\infty}^{t} C \cdot v\frac{dv}{dt}\,dt = C\int_0^{t} v\,dv$$

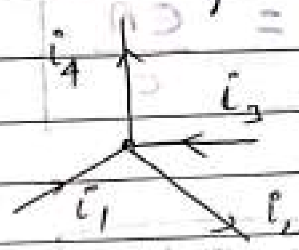
$$\boxed{W_c = \frac{1}{2}CV^2}$$

Kirchhoff's laws & Resistive Networks! –

"Current Law" (KCL) ! –

"किसी node पर सभी आने और जाने वाली सभी धाराओं का योग 0 होता है"।

$$\sum i = 0.$$



∴ Incoming current = Outgoing current

$$\left[i_1 + i_3 = i_2 + i_4 \right]$$

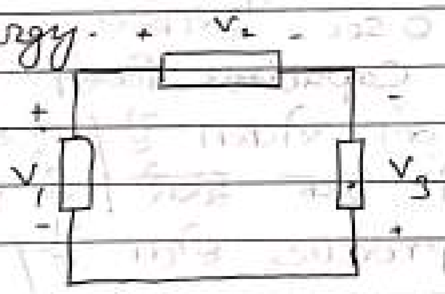
$$i_1 + i_2 + i_3 + i_4 = 0.$$

- law is based on law of conservation of charge.

- Kirchhoff's Voltage law (KVL) ! –

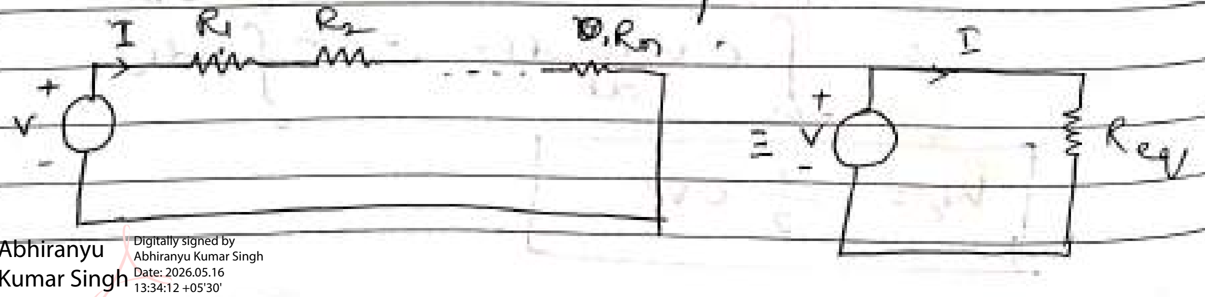
"किसी बंद परिपथ में सभी voltages का योग 0 होता है"।

- law is based on law of conservation of energy.



$$V_1 = V_2 - V_3.$$

- Resistor in Series :– यदि सारे element जिसमें same current flow होता है, Series connection कहलाता है।



$V_1 + V_2 + \cdots\cdots V_n - V = 0.$

$V_1 = IR_1, \quad V_2 = IR_2, \cdots V_n = IR_n.$

$IR_1 + IR_2 + \cdots + IR_n = V$

$$I = \frac{V}{R_1 + R_2 + \cdots + R_n}$$

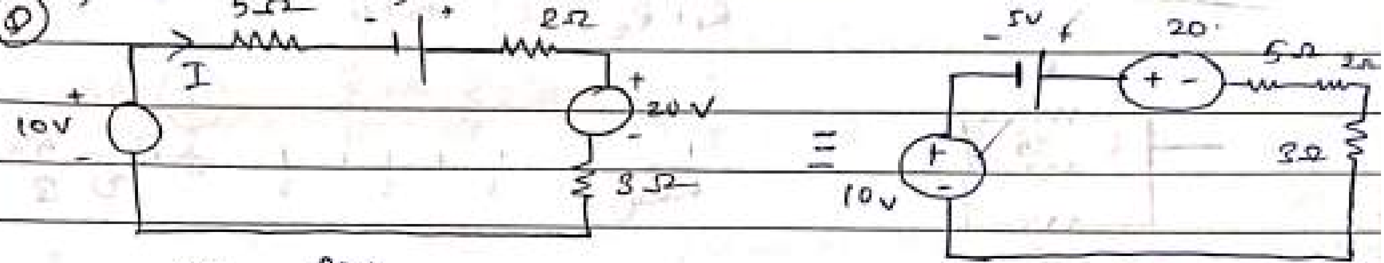
For equivalent N/W

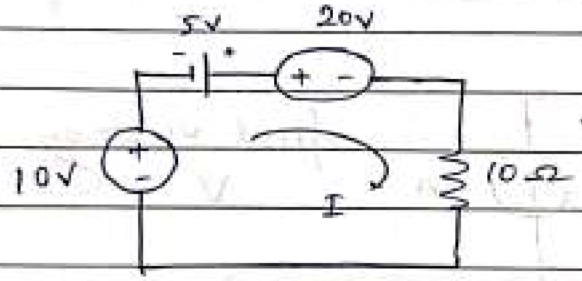
$$I = \frac{V}{R_{eq}}$$

Comparing

$$\therefore \boxed{R_{eq} = R_1 + R_2 + \cdots\cdots + R_n}$$

① find I:



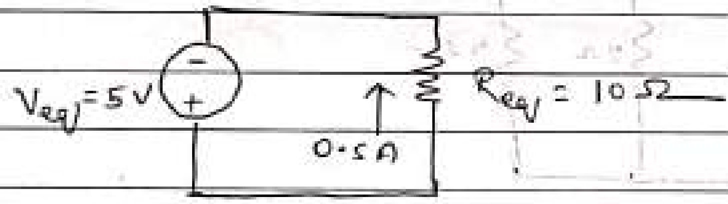


→ Apply KVL :-

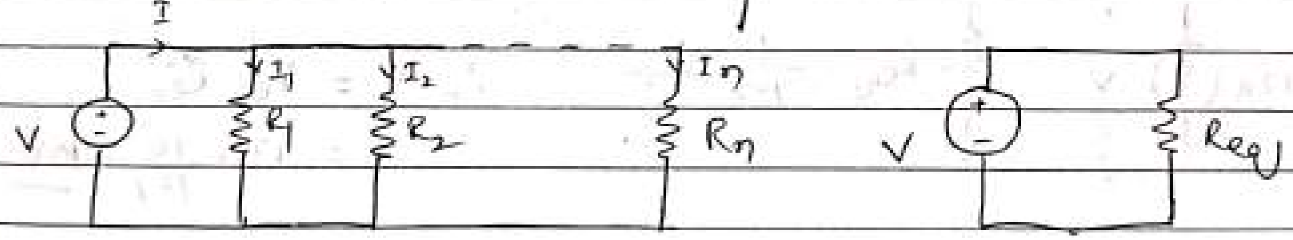
$-5 + 20 + 10I = 10$

$10I = -5$

$I = -\frac{1}{2}\ A.$



- Resistors in Parallel :— सभी Network element Parallel में connect होंगे यदि उनके Across का Voltage समान होगा।



$I = I_1 + I_2 + \cdots\cdots + I_n$

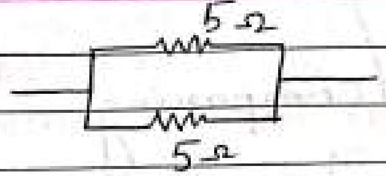
but $I_1 = \frac{V}{R_1}, \quad I_2 = \frac{V}{R_2} \quad \cdots \quad I_n = \frac{V}{R_n}$

$\therefore I = \frac{V}{R_1} + \frac{V}{R_2} + \cdots\cdots + \frac{V}{R_n}$

$I = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots\cdots + \frac{1}{R_n}\right)$

$$I = \frac{V}{R_{eq}}$$

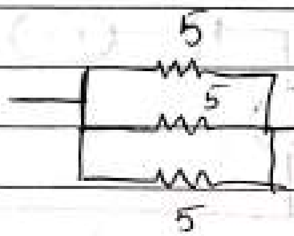
$$\therefore \boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}}$$ Resistance in Parallel



$$\frac{1}{R_{eq}} = \frac{1}{5} + \frac{1}{5} = \frac{2}{5}$$

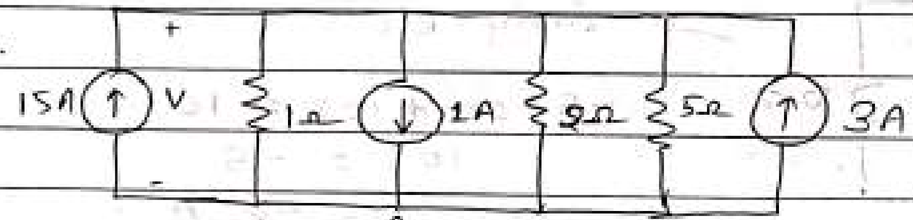
$$\therefore R_{eq} = 2.5\,\Omega$$

Alt :- $R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{5 \times 5}{5+5} = 2.5\,\Omega$

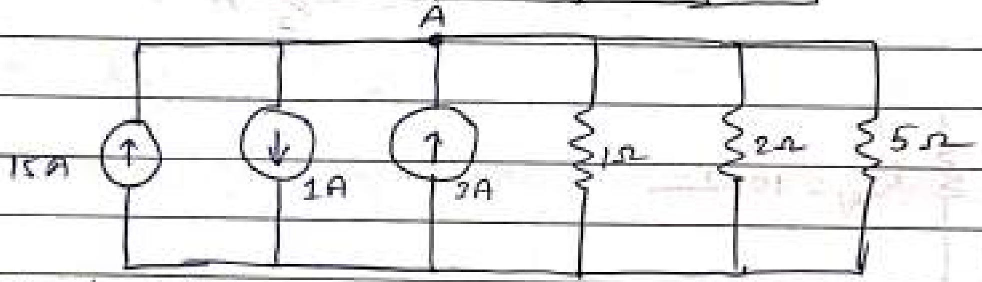


$$\frac{1}{R_{eq}} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} \Rightarrow R_{eq} = \frac{5}{3}\Omega$$

Q.

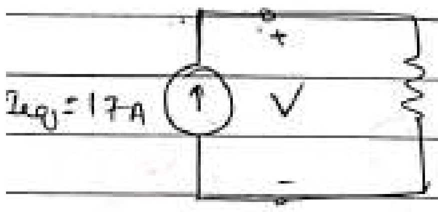


find voltage V



KCL at node A, $I_{eq} = 15 + 3 - 1 = 17\text{ A}$.

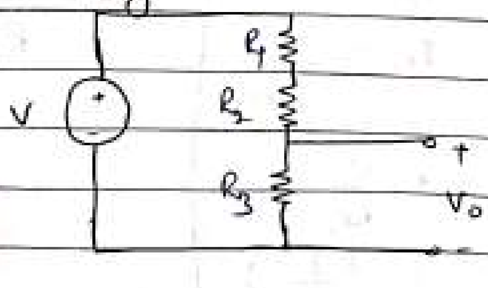
$$\frac{1}{R_{eq}} = \frac{1}{1} + \frac{1}{2} + \frac{1}{5} = 1.7\,\Omega$$



$R_{eq} = \frac{1}{1.7}\,\Omega$

$$\therefore V = I_{eq} R_{eq}$$
$$= 17 \times \frac{10}{17} = 10\text{V}$$

- Voltage divider :-



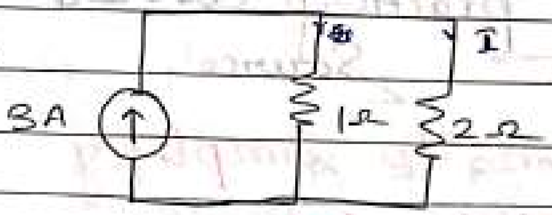
$$\boxed{V_o = \frac{R_3}{R_1 + R_2 + R_3} V}$$

• Current divider :-

$$I_1 = I \times \frac{R_2}{R_1 + R_2}$$

$$I_2 = I \times \frac{R_1}{R_1 + R_2}$$

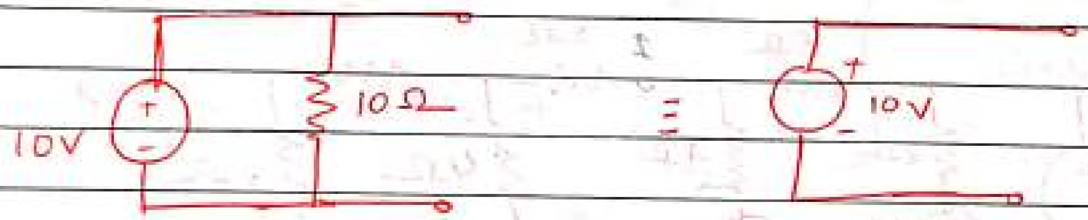
Q.

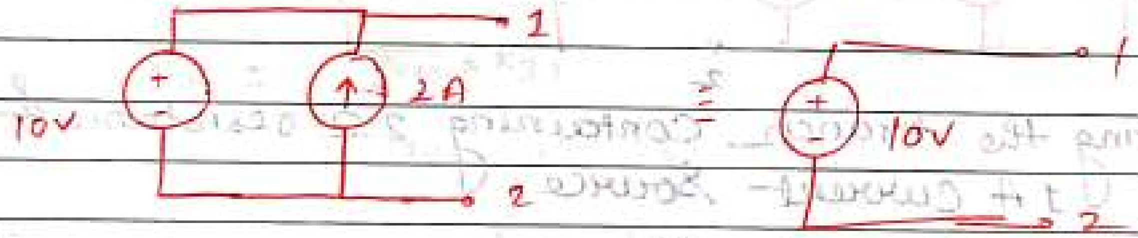


find Current I ?

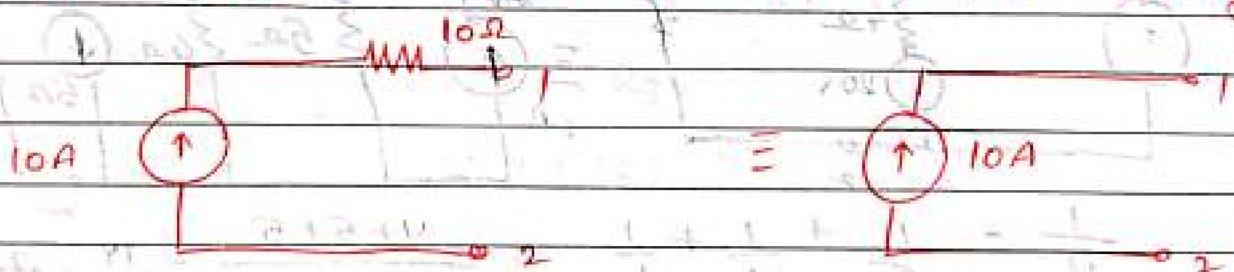
$$I = \frac{1}{1+2} \times 3 = 1\ A$$

• Voltage Source with parallel resistors or current source

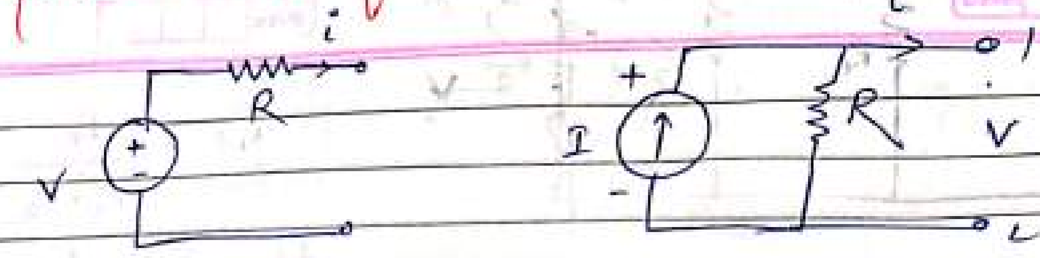




• Current Source in Series with resistors or Voltage Source

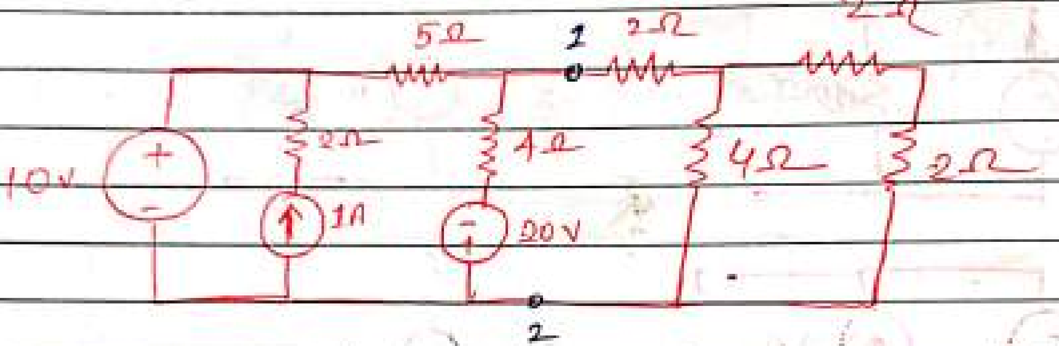


Source Transformation :-

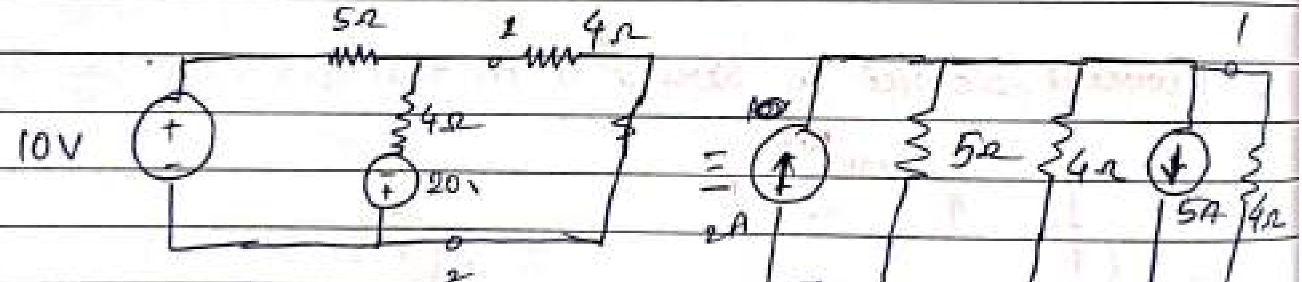


practical voltage Source — practical current Source.

Q. Use Source Transformation to Simplify N/W and find equivalent network Containing only voltage Source & resistance at terminal 1, 2.



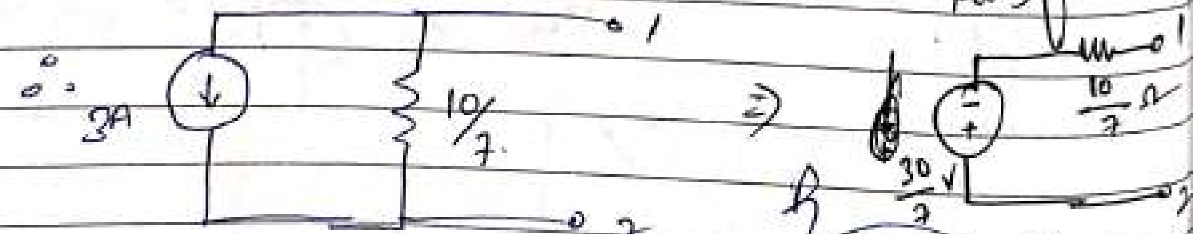
Ignoring the branch Containing 2Ω resistance & 1A current Source.



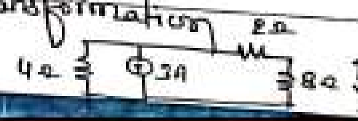
$$\frac{1}{R} = \frac{1}{5} + \frac{1}{4} + \frac{1}{4} = \frac{4+5+5}{20} = \frac{14}{20} = \frac{7}{10}$$

$$R = \frac{10}{7}\ \Omega$$

Source transformation



Q. दिए गए circuit में source transformation की मदद से V_0 ज्ञात करें।



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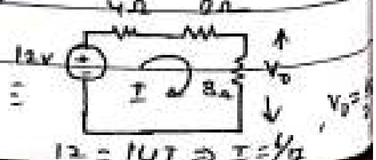
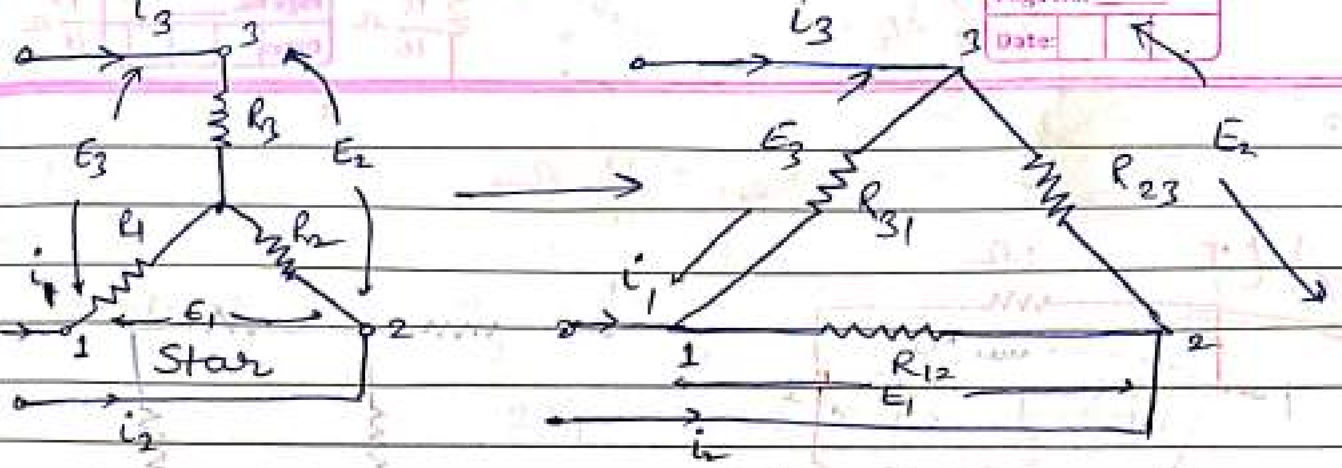


$12 = 14I \Rightarrow I = 6/7$

• Star to Delta or Y – Δ transformation :–



$$R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

$$R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1}$$

$$R_{13} = R_1 + R_3 + \frac{R_1 R_3}{R_2}$$

Star to Delta.

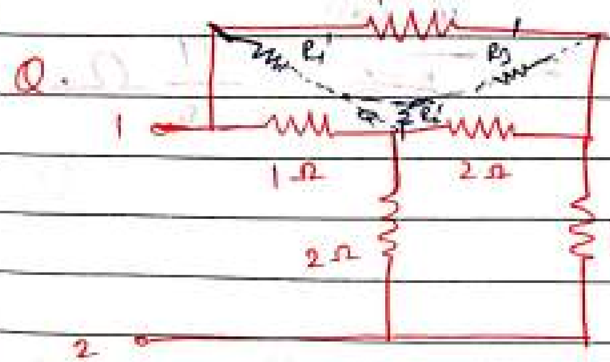
Delta to star :–

$$R_1 = \frac{R_{12} \times R_{31}}{R_{12} + R_{23} + R_{31}}$$

$$R_2 = \frac{R_{23} \cdot R_{12}}{R_{12} + R_{23} + R_{31}}$$

$$R_3 = \frac{R_{23} \cdot R_{31}}{R_{12} + R_{23} + R_{31}}$$

Q. find equivalent Resistance b/w terminal 1 & 2.



$$R_1' = \frac{1 \times 1}{1+1+2} = \frac{1}{4}\,\Omega$$

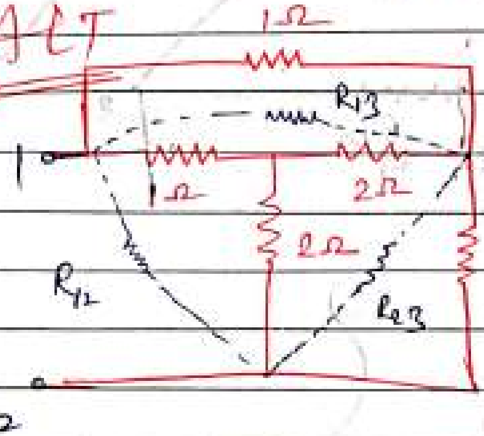
$$R_2' = \frac{1 \times 2}{1+1+2} = \frac{2}{4}\,\Omega = \frac{1}{2}\,\Omega$$

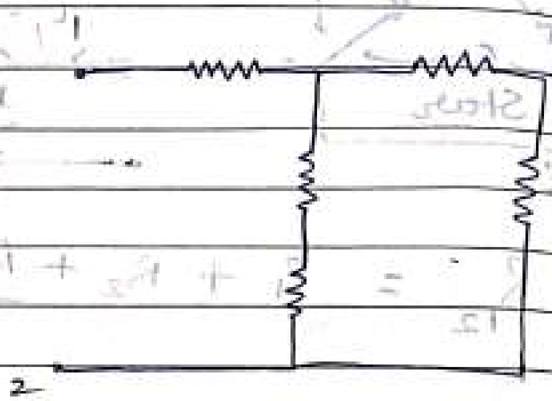
$$R_3' = \frac{2 \times 1}{1+1+2} = \frac{2}{4} = \frac{1}{2}\,\Omega$$

$= \frac{48}{7} V$

$$\therefore R_{eq} = \frac{19}{16}\,\Omega$$

ACT

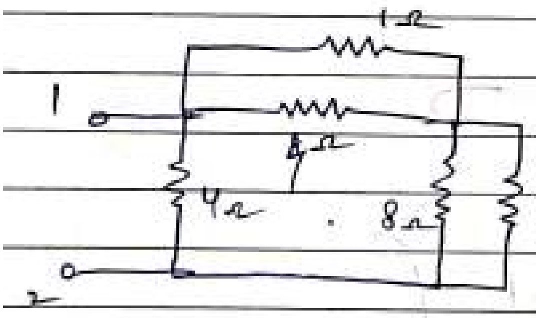


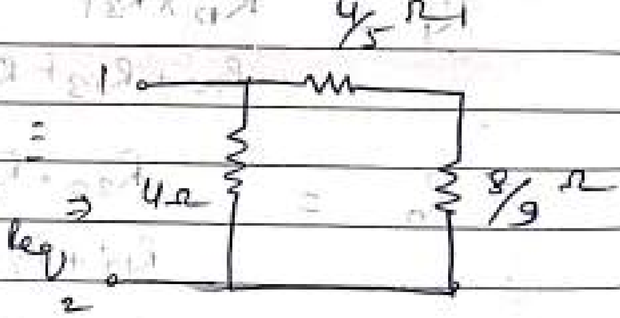


$$R_{12} = \cancel{\frac{1\times 2}{1+2+2} = \frac{2}{5}\,\Omega} = 1+2+\frac{1\times 2}{2} = 4\,\Omega$$

$$R_{23} = 2+2+\frac{2\times 2}{1} = 8\,\Omega$$

$$R_{13} = 1+2+\frac{1\times 2}{2} = 4\,\Omega$$

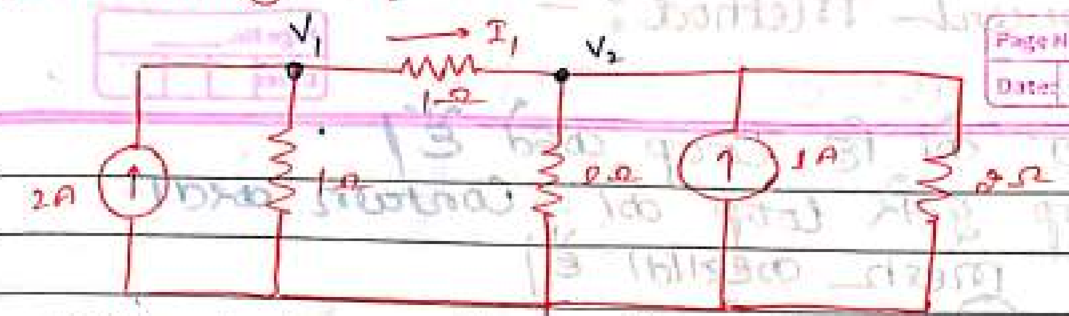




$$R_{eq} = \frac{4}{5} + \frac{8}{9} = \frac{36+40}{45} = \frac{76}{45}\,\Omega$$

$$\therefore R_{eq} = \frac{4\times 76/45}{4+76/45} = \frac{\cancel{4}\times 76}{\cancel{256}\;64} = \frac{19}{16}\,\Omega$$

Node Voltage Method :-



Node at V_1, $\frac{V_1}{1} + \frac{V_1 - V_2}{1} = 2$

$2V_1 - V_2 = 2$ —— (i)

Node at V_2, $\frac{V_2 - V_1}{1} + \frac{V_2}{2} + \frac{V_2}{2} = 1$

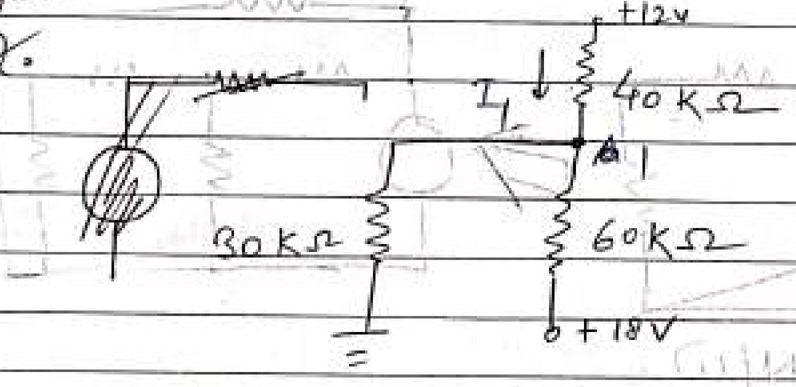
~~$-2V_1 + 2V_2$~~ $-V_1 + 2V_2 = 1$ —— (ii)

$V_2 = \frac{4}{3} V_1$ $\quad V_1 = 5/3 \text{ V}$.

$I_1 = \frac{V_1 - V_2}{1} = \frac{5}{3} - \frac{4}{3} = 1/3 \text{ A}$

Exam

Q.



find the Current in R_1 resistance!

$$\frac{V_1 - 12}{40K\Omega} + \frac{V_1 - 18}{60K} + \frac{V_1}{30K} = 0.$$

$$\frac{V_1 - 12}{4} + \frac{V_1 - 18}{6} + \frac{V_1}{3} = 0.$$

3, 4, 6

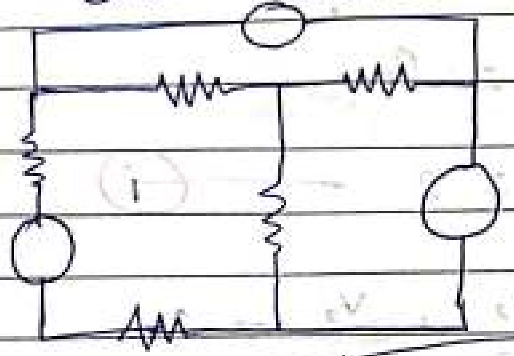
$$\frac{3V_1 - 36 + 2V_1 - 36 + 4V_1}{12} = 0 \Rightarrow 9V_1 = 72$$

$V_1 = 8 \text{ V}$

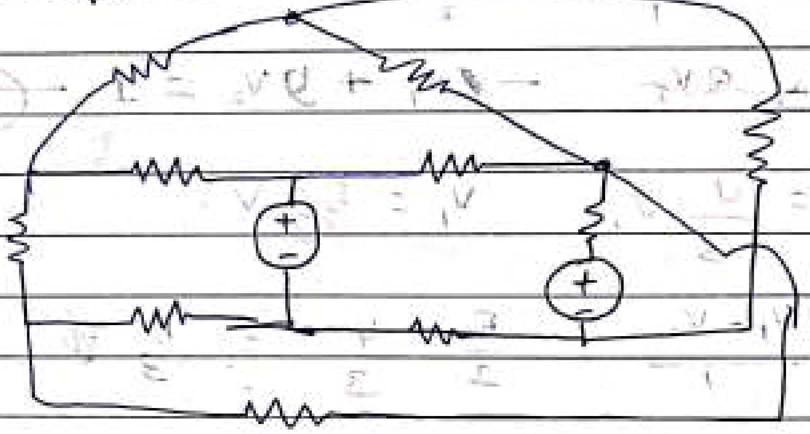
$\therefore I_1 = \frac{12 - 8}{40K\Omega} = 0.1 \text{ mA}$ - Ans

Mesh Current Method :-

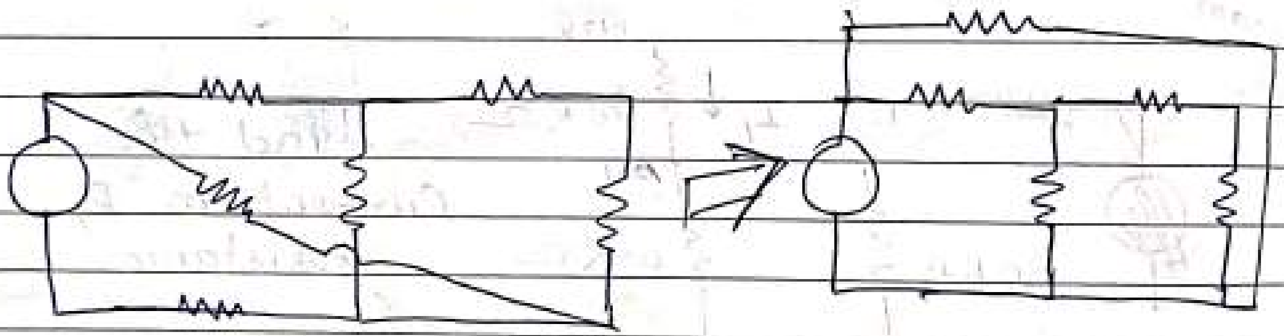
- बंद परिपथ को हि Loop कहते हैं।
- एक Loop दूसरे Loop को Contain करता है तो mesh कहलाता है।



→ Planer N/W



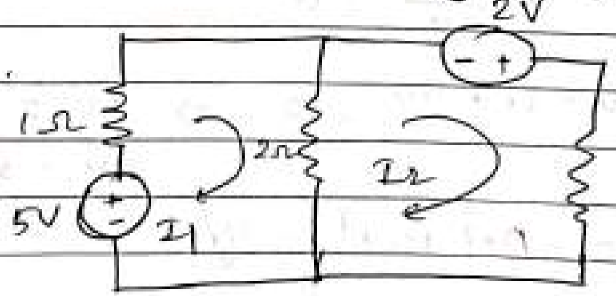
→ Non-planer



Planer N/W.

- Planer N/W वैसे N/W होते हैं जिसे हम एक Paper पर Draw कर सकते हैं बिना किसी lines के Crossing के।

Q.



find Current in 2Ω resistor?

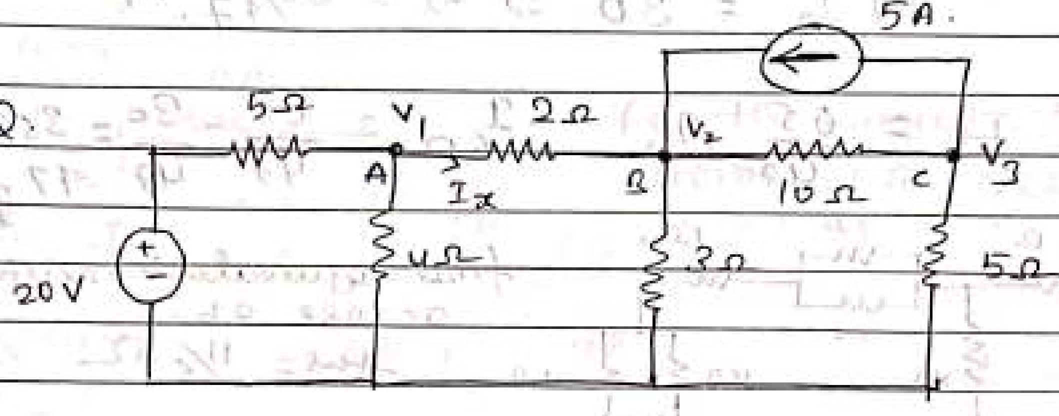
$I_1(1) + (I_1 - I_2) \times 2 = 5 \Rightarrow 3I_1 - 2I_2 = 5$ — (i)

$I_2(1) + (I_2 - I_1) \times 2 = 2 \Rightarrow -2I_1 + 3I_2 = 2$ — (ii)

$\therefore I_1 = \frac{19}{5} A, \quad I_2 = 16/5 A.$

$\therefore$ Current in 2Ω resistor $= I_1 - I_2 = 3/5 A$

Q.



find I_x

Node at A :-
$$\frac{V_1 - 20}{5} + \frac{V_1 - V_2}{2} + \frac{V_1}{4} = 0.$$

2 | 2, 4, 5
1, 2, 5

$$\frac{4V_1 - 80 + 10V_1 - 10V_2 + 5V_1}{20} = 0.$$

$$19V_1 - 10V_2 = 80 \text{ — (i)}$$

Node at B :-
$$\frac{V_2 - V_3}{10} + \frac{V_2 - V_1}{2} + \frac{V_2}{3} = 3$$

$$\frac{3V_2 - 3V_1 + 2V_2}{6} = 3.$$

$$-3V_1 + 5V_2 = 18 \text{ — (ii)}$$

Node
$$\frac{3V_2 - 3V_3 + 15V_2 - 15V_1 + 10V_2}{30} = 3.$$

$$-15V_1 + 28V_2 - 3V_3 = 90 \text{ — (ii)}$$

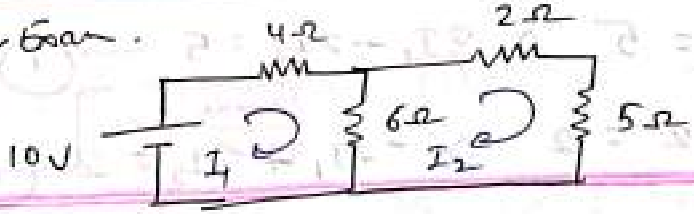
Node at C :-
$$\frac{V_3 - V_2}{10} + \frac{V_3}{5} + 5 = 0.$$

$$\frac{V_3 - V_2 + 2V_3 + 50}{10} = 0.$$

$$-V_2 + 3V_3 + 50 = 0 \text{ — (iii)}$$

$\therefore \boxed{I_x = 0.08 A}$

Ques.



find current in 6Ω resistor?

$$10 = 10I_1 - 6I_2 \quad \text{(i)} \times 3$$

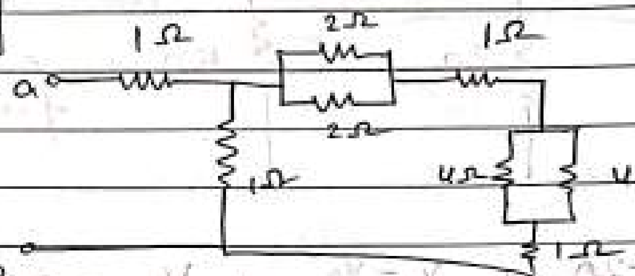
$$0 = -6I_1 + 13I_2 \quad \text{(ii)} \times 5$$

$$\therefore 47I_2 = 30 \Rightarrow I_2 = 30/47 \text{ A}$$

$$\therefore I_1 = \frac{65}{47} \text{ A}, \quad I_{6\Omega} = \frac{65}{47} - \frac{30}{47} = \frac{35}{47} \text{ A}$$

H.W

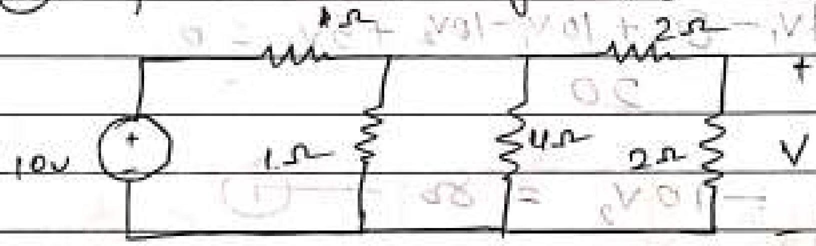
(1)



find equivalent resistance across ab.

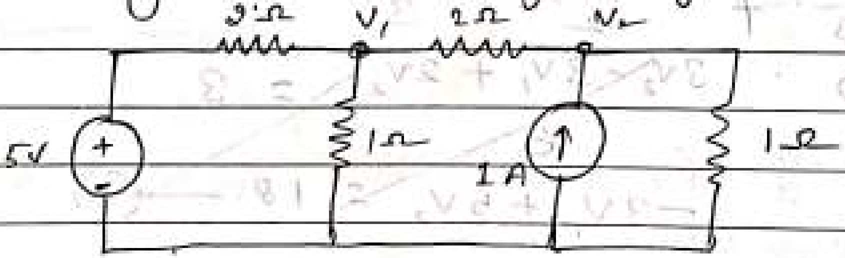
Ans = 11/6 Ω

(2) find the voltage 'V'



Ans = 2V

(3) Using nodal Analysis find voltages V_1 & V_2.

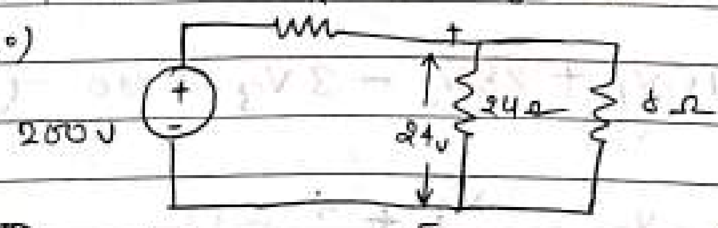


$V_1 = \frac{17}{11}$ V

$V_2 = \frac{13}{11}$ V

(4) find the value of R in given circuit.

(IES EC 2010)



(a) 4Ω (b) 40Ω (c) 44Ω (d) 440Ω

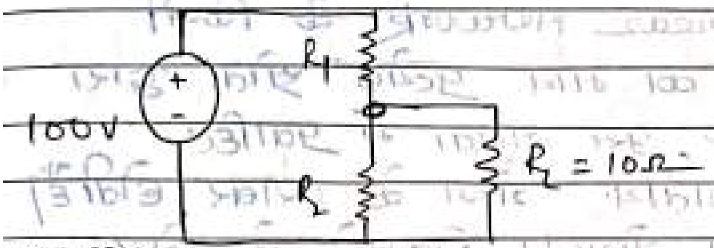
Q

find equivalent resistance across ab

(a) 2Ω (b) 3Ω (c) 1Ω (d) 0.5Ω

(5) What is the voltage across load resistance R_L in the above circuit ? The value of each resistor connected in the circuit is 10Ω

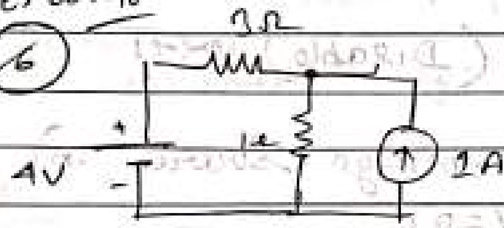
IES 2014



(a) 3.33V (b) 33.33V (c) 333.33V (d) 0V

IES EE 2010

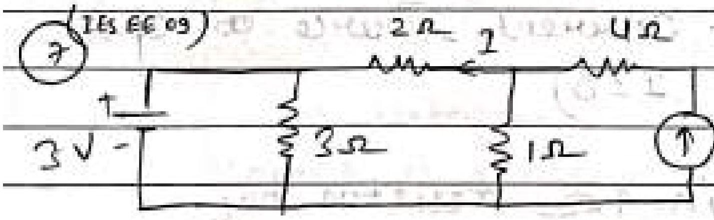
(6)



For the circuit, the voltage across 1Ω resistor is

(a) $7/4$ V (b) $5/4$ V (c) $7/3$ V (d) $2/3$ V

(7) (IES EE 09)

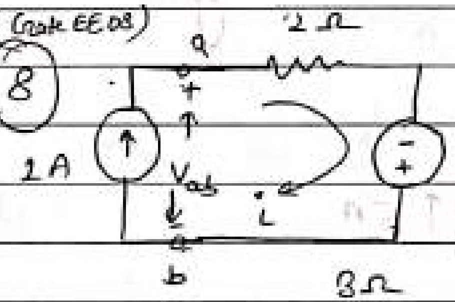


What is the value of I

(a) 4A (b) 3A (c) 2A (d) 1A

(Gate EE 08)

(8)

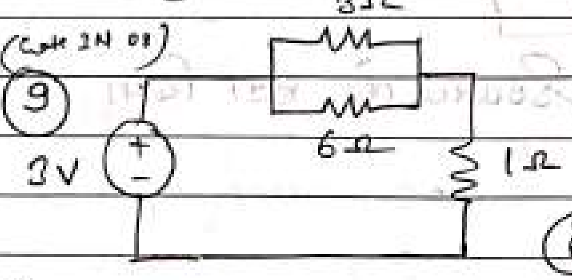


The voltage V_{ab} will be :-

(a) −3V (b) 0 V (c) 3V (d) 5V

(Gate IN 08)

(9)

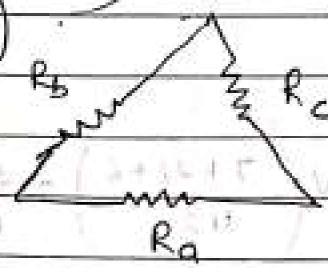


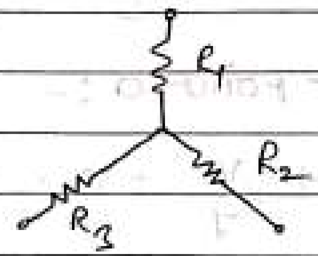
Power supplied by DC voltage source in circuit.

(a) 0 W (b) 1 W (c) 2.5 W (d) 3 W

(Gate EE 04)

(10)





If $R_a = 20\Omega$, $R_b = 10\Omega$, $R_c = 10\Omega$. The resistance R_1, R_2 & R_3 in Ω of an equivalent star connection

(a) 2.5, 5, 5 (b) 5, 2.5, 5

(c) 5, 5, 2.5 (d) 2.5, 5, 2.5

Network theorems :-

(1) Superposition Theorem :- इस Theorem के अनुसार "किसी Linear Network के किसी शाखा में कुल धारा का मान प्रत्येक स्रोत द्वारा अकेले कार्य करते हुए, उस शाखा में प्रवाहित धाराओं के बीजगणितीय योग के बराबर होती है

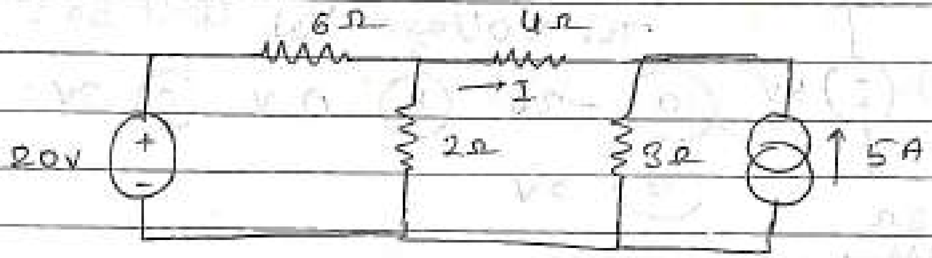
किसी एक Source का योगदान निकालने के लिए अन्य Source को निष्क्रिय (Disable) करना आवश्यक है, इसके लिए

(a) सभी अन्य independent Voltage Source को Short करना पड़ता है। (V=0)

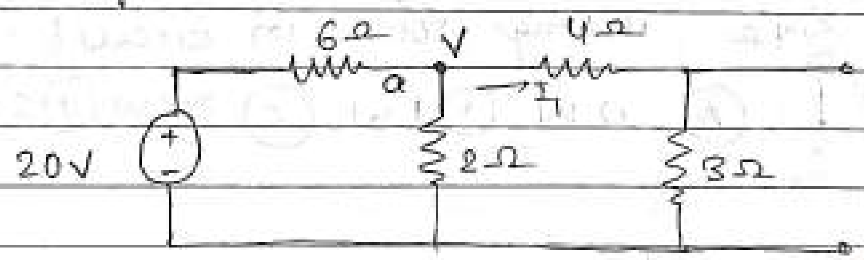
(b) सभी independent current Source को open करना पड़ता है। (I=0)

(Dec 11-12)

(1) find the current in 4Ω resistor by Superposition Theorem.



Step I :- ~~किसी~~ Current Source को हटा दिया



Node analysis at point a :-

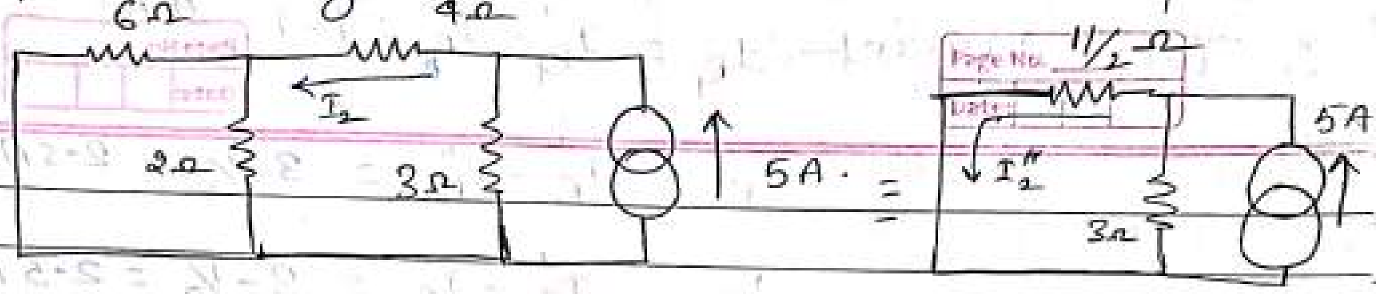
$$\frac{V-20}{6} + \frac{V}{2} + \frac{V}{7} = 0 \Rightarrow V\left(\frac{7+21+6}{42}\right) = \frac{20}{6}$$

$$V = \frac{70}{17} V$$

$$\therefore \text{Current in } 4\Omega = I_1' = \frac{70}{17 \times 7} = \frac{10}{17} A$$

L(1)

Step-II :- Voltage Source को remove करना /



$$I_2 = 5 \times \frac{3}{3+1\frac{1}{2}} = \frac{5\times 6}{17} = \frac{30}{17} A.$$

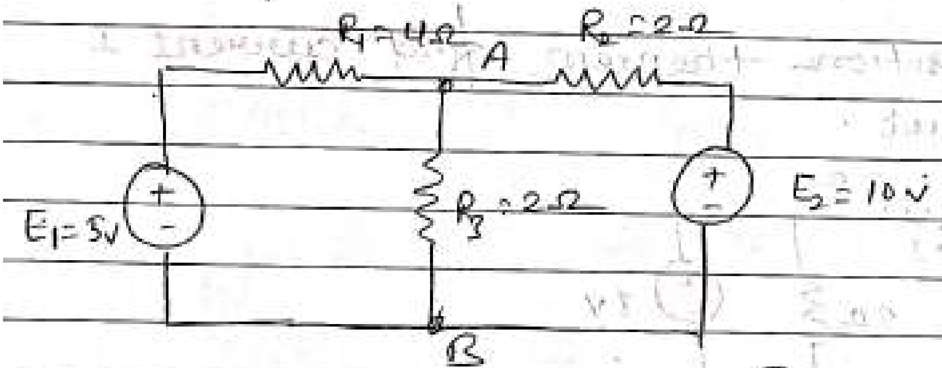
Step-III :-

$$\therefore I_{Net} = I_2 - I_1 = \frac{30}{17} - \frac{10}{17} = \frac{20}{17} A.$$

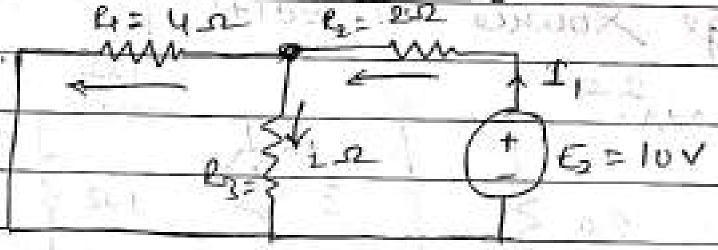
Direction opposite $= -20/17 A$

Dec-2011-12

(2) Using Superposition theorem, determine the current flowing through resistors R_1, R_2 & R_3 of the network. Also find the potential of point A relative to B.



Step-I :- $E_1 = 5V$ voltage Source को ECT किया | ~~और~~



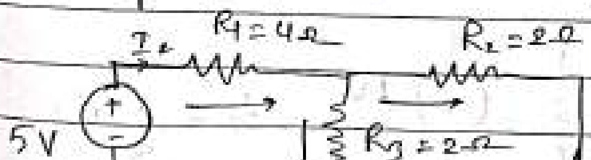
$$I'_{R_2} = 3A.$$

$$I'_{R_1} = 3 \times \frac{2}{6} = 1A.$$

$$I_1 = \frac{10}{4/3 + 2} = 3A.$$

$$I'_{R_3} = 3 \times 4/6 = 2A.$$

Step-II :- $E_2 = 10V$ ~~V~~oltage Source को ECT किया



$$\therefore I''_{R_1} = 1A.$$

$$I''_{R_2} = 1 \times \frac{2}{4} = \frac{1}{2} A.$$

$$I_2 = \frac{5}{4+1} = 1A.$$

$$I''_{R_3} = 1 \times 2/4 = \frac{1}{2} A.$$

Step-III :-

$\therefore$ Total current $I_{R_1} = I'_{R_1} - I''_{R_1} = 1-1 = 0\,A$

$$I_{R_2} = I'_{R_2} - I''_{R_2} = 3 - \tfrac{1}{2} = 2.5\,A$$

$$I_{R_3} = I'_{R_3} - I''_{R_3} = 3 - \tfrac{1}{2} = 2.5\,A.$$

$$V_{AB} = R_3 \times I_{R_3} = 2 \times 2.5 = 5\,V$$

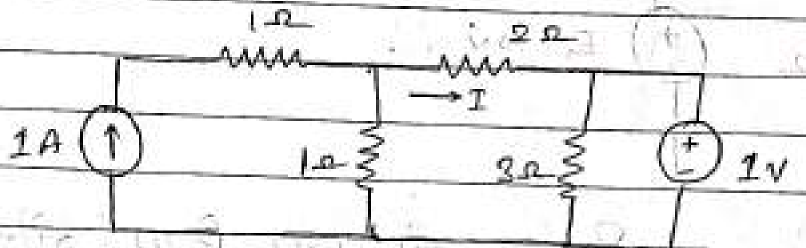
Proof: V_{AB} : $\dfrac{V-5}{4} + \dfrac{V}{2} + \dfrac{V-10}{2} = 0.$

$$\frac{V-5+2V+2V-20}{4} = 0.$$

$$5V = 25 \Rightarrow \boxed{V = 5\,V}$$

Dec-2010-11

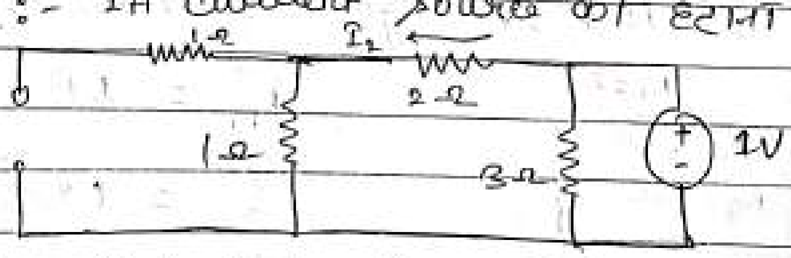
Q(1) Using superposition theorem, find current I in the circuit.



Case-I :- 1V Voltage Source को हटा ।

$$\therefore I_1 = 1 \times \frac{1}{1+2} = \frac{1}{3}\,A$$

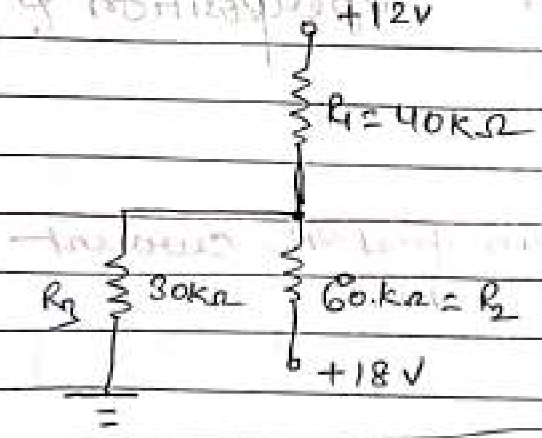
Case-II :- 1A current Source को हटा



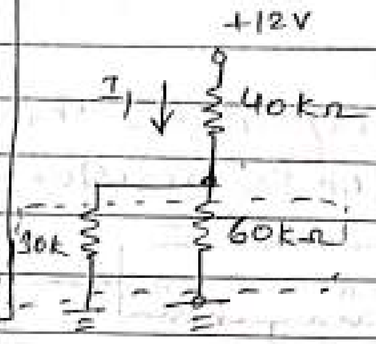
$I_2 = \dfrac{1}{3}\,A.$

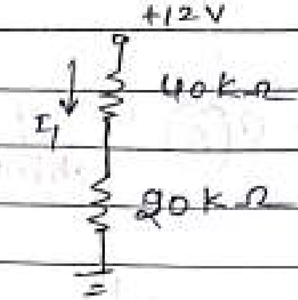
Case-III :- Net current $= I_1 - I_2 = \tfrac{1}{3} - \tfrac{1}{3} = 0\,A.$

(4) Q. Use Superposition theorem to find the current through R_1 in the circuit.



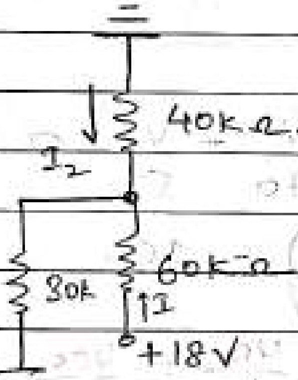
Case-I :- 18V voltage को remove करो।





$\therefore \; I_1 = \frac{12}{60} mA = \frac{1}{5} mA$ — (i)

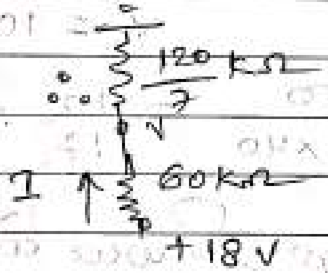
Case-II :- 12V voltage source को remove करो।



(a)

$\therefore$ 30kΩ & 40kΩ resistance are in parallel combination,

So, $R_{eq} = \frac{40 \times 30}{70} k\Omega = \frac{120}{7} k\Omega$



$\therefore V = \frac{18 \times \frac{120}{7}}{60 + \frac{120}{7}} = \frac{18 \times 4}{\frac{540}{30}}$

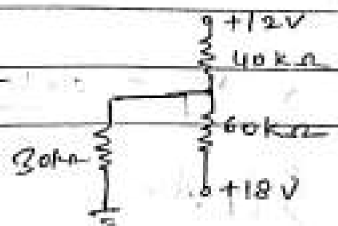
$= \frac{7}{30} mA \cdot 4V$

$\therefore I.$

Now from figure (a) :- $I_2 = \frac{7}{30} \times \frac{30}{70} = \frac{1}{10} mA$.

$\therefore$ Case-III :- Total current through 40kΩ resistor $= I_1 - (-I_2) = \frac{7}{30} - \frac{1}{10} = \frac{1}{10}$

Proof :-



$\frac{V-12}{40} + \frac{V-18}{60} + \frac{V}{30} = 0.$

$9V = 72 \Rightarrow \boxed{V = 8V}$

$I_{40k\Omega} = \frac{12-8}{40} = \frac{1}{10} mA$

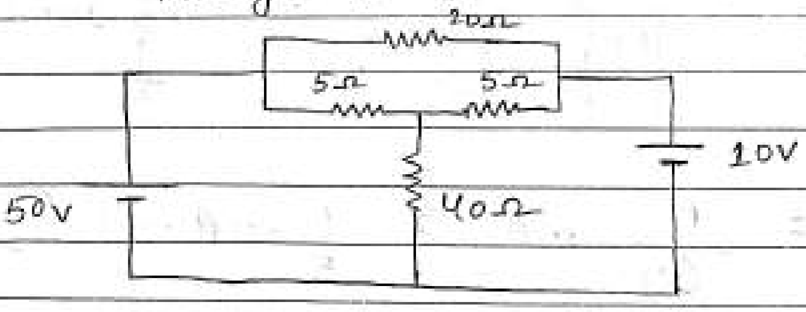
Q5 

find current in 2Ω using Superposition ?

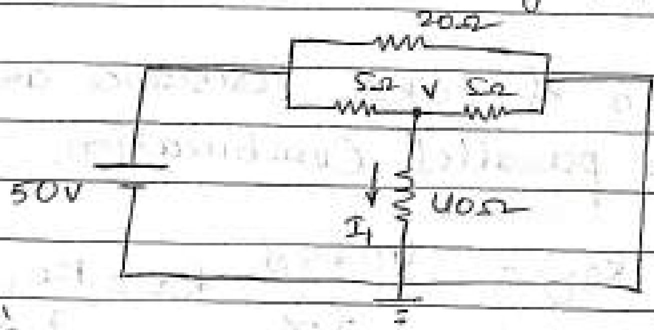
(Ans = 21/8 A)

(Nov-Dec 15)

Q6 Using superposition theorem, find the current through 40Ω resistor in ckt.



Case-I:- 10 V voltage source को remove करते हैं



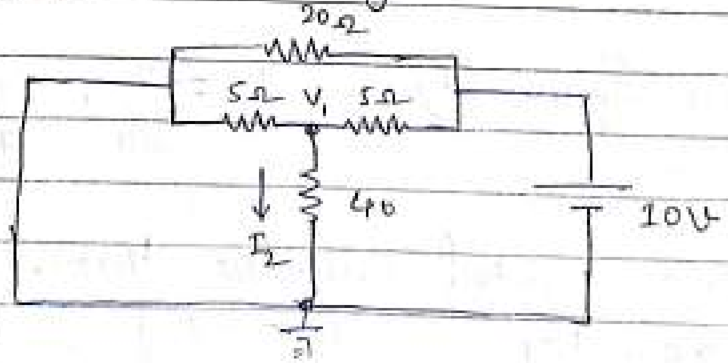
$$\frac{V-50}{5} + \frac{V}{40} + \frac{V}{5} = 0$$

$$V\left(\frac{2}{5} + \frac{1}{40}\right) = 10$$

$$V = 10 \times \frac{40}{17} = \frac{400}{17}$$

$$\therefore I_1 = \frac{400}{17 \times 40} = \frac{10}{17}\,A$$

Case-II:- 50 V voltage source को remove करते हैं



$$\frac{V_1 - 10}{5} + \frac{V_1}{40} + \frac{V_1}{5} = 0$$

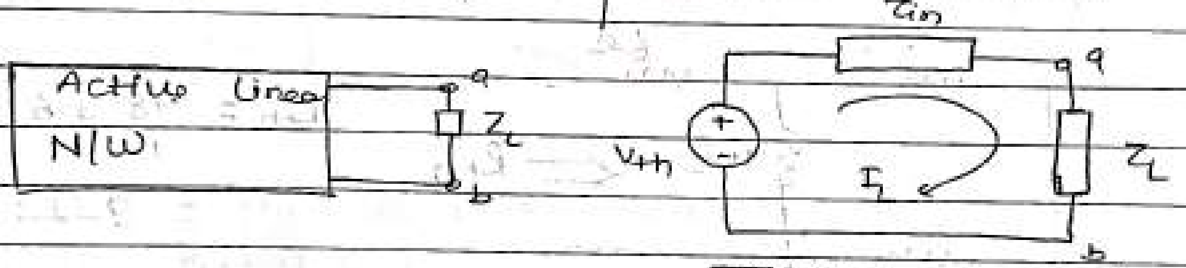
$$V_1\left(\frac{2}{5} + \frac{1}{40}\right) = 2 \Rightarrow V_1 = \frac{2 \times 40}{17} = \frac{80}{17}$$

$$\therefore I_2 = 2/17\,A$$

Case-III :- Total current $= \frac{10}{17} + \frac{2}{17} = \frac{12}{17}$ A Ans

II. Thevenin's theorem :—

इस Theorem के अनुसार voltage source, current source एवं impedance से निर्मित किसी भी linear circuit का इनके किन्ही दो Terminal के बीच का व्यवहार एक तुल्य voltage source V_{th} एवं equivalent Impedance (R_{th}) के द्वारा दिखाया जाता है।

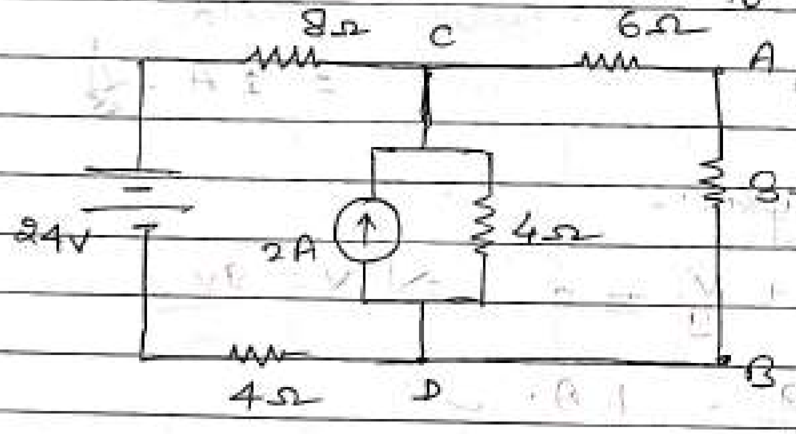


Thevenin equivalent

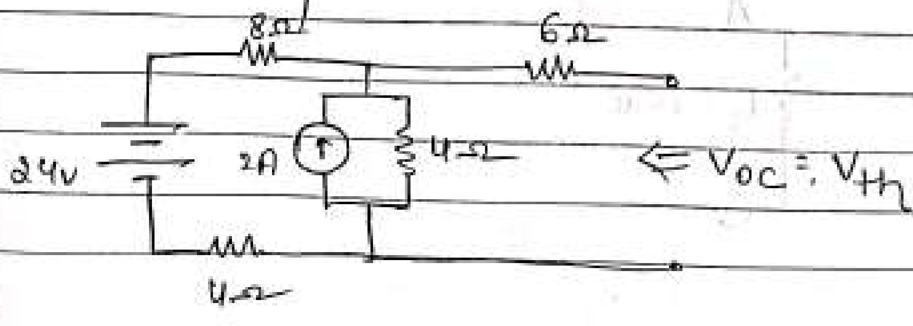
$$\therefore \boxed{I_L = \frac{V_{th}}{Z_{in} + Z_L}}$$

Exam 2012

(1) With the help of thevenin's theorem. Calculate current flowing through 8Ω res



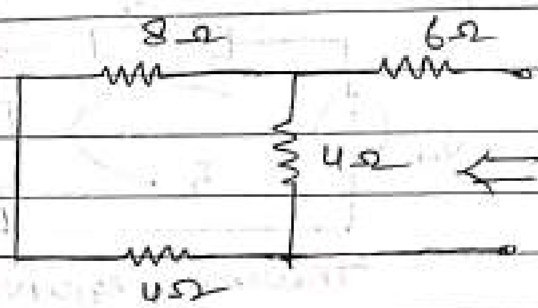
Step-I :- Open circuit voltage (V_{OC}) को पता करना। इसके लिए जिस electronic component के Across current / voltage निकालना है, उसे open कर देते हैं।



$$\frac{V_{th}-24}{12} + \frac{V_{th}}{4} = 2 \Rightarrow \frac{V_{th}-24+3V_{th}}{12} = 2$$

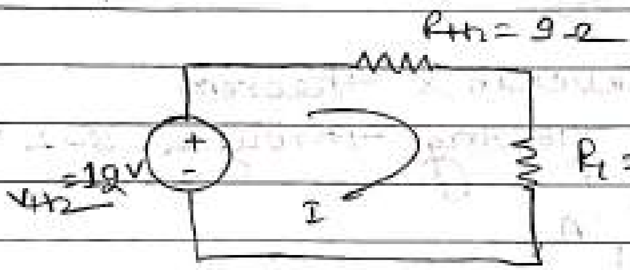
$$\therefore \boxed{V_{th} = 12V}$$

Step-2 :- Thevenin equivalent resistance (R_{th}) independent को find करना। इसके लिए Voltage source को short circuit और independent current source को open circuit करते हैं।



$$R_{th} = \frac{48}{16} + 6$$
$$= 9\Omega$$

Step 3 :- Thevenin equivalent circuit model.



$$\therefore I_{3\Omega} = \frac{12}{12} A$$
$$= 1 A.$$

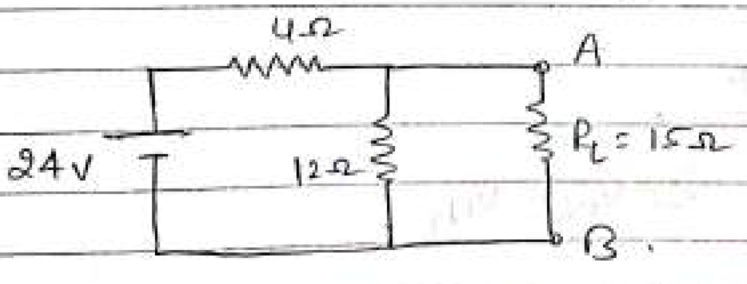
proof :- Node Analysis :-

$$\frac{V-24}{12} + \frac{V}{4} + \frac{V}{9} = 2 \Rightarrow \boxed{V = 9V}$$

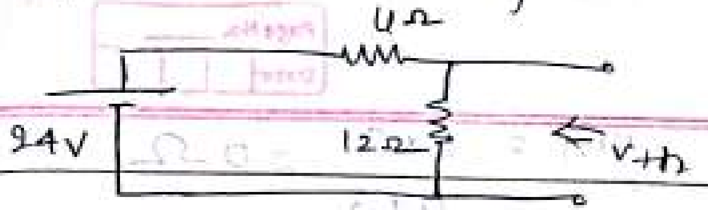
$$\therefore I_{3\Omega} = \frac{9}{9} = 1 A.$$

Dec 2011-12

(2) Using thevenin's theorem, find the current in load resistor $R_L = 15\Omega$.



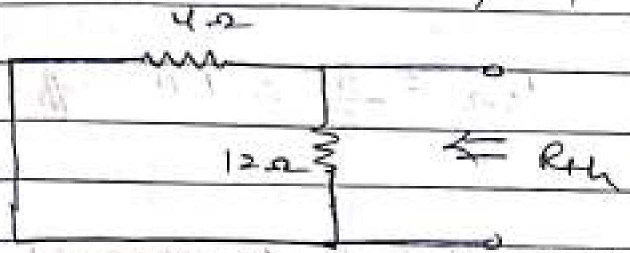
Step-I :- V_{th} को find करना |



$$\therefore V_{th} = \frac{24 \times 12}{16} = \frac{6 \times 12}{4} \cdot \frac{3}{} $$

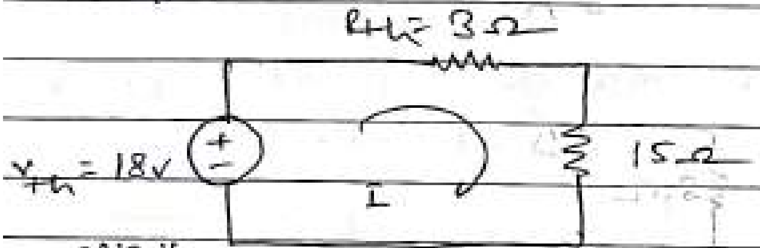
$$= 18\,V.$$

Step-II :- R_{th} को find करना |



$$R_{th} = \frac{4 \times 12}{16} = 3\Omega$$

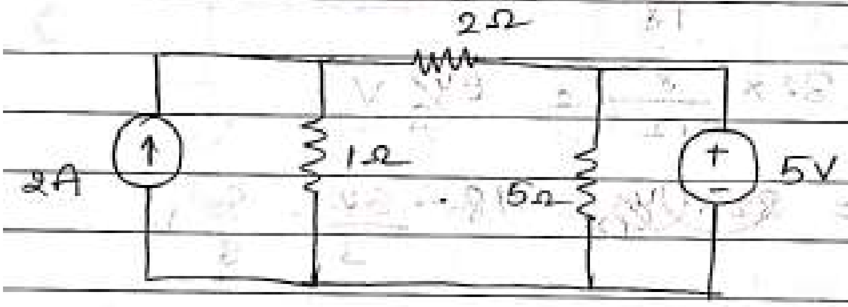
Step-III :- Thevenin equivalent model :-



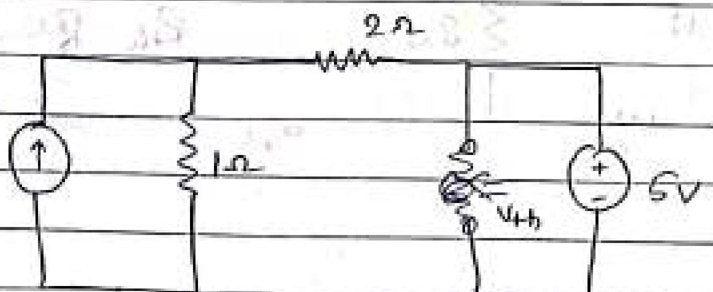
$$I_{15\Omega} = \frac{18}{18} = 1\,A$$

Dec 2010-11

(3) find the current through the 5Ω resistor in the circuit

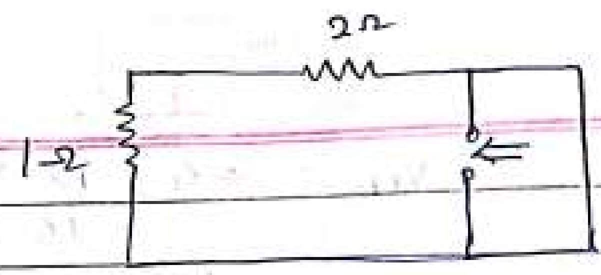


Step-I :- V_{th} को find करना |



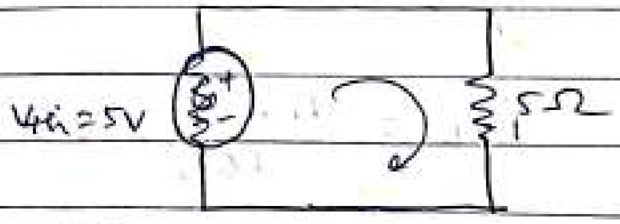
$$\therefore V_{th} = 5V$$

Step-II :- Thevenin equivalent resistance (R_{th}) को find करना है |



$$R_{th} = \frac{0\times 3}{0+3} = 0\ \Omega$$

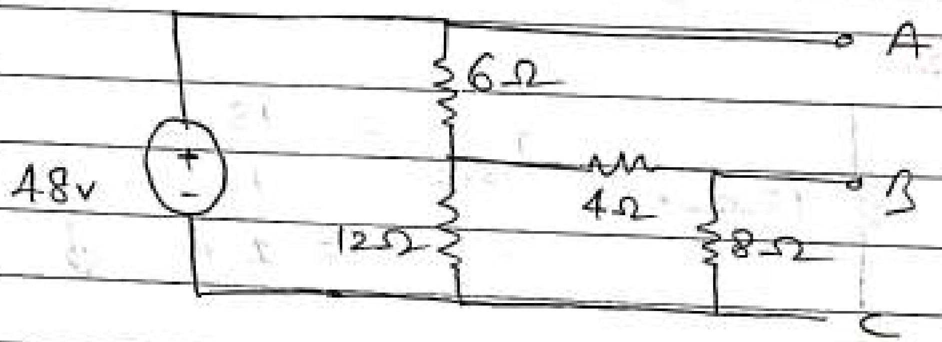
Step. 3:- Thevenin equivalent model :-



$$I_{5\Omega} = \frac{5}{5} = 1\ A$$

Dec. 2015

Q. find R_{th} & V_{th} b/w A & B terminal



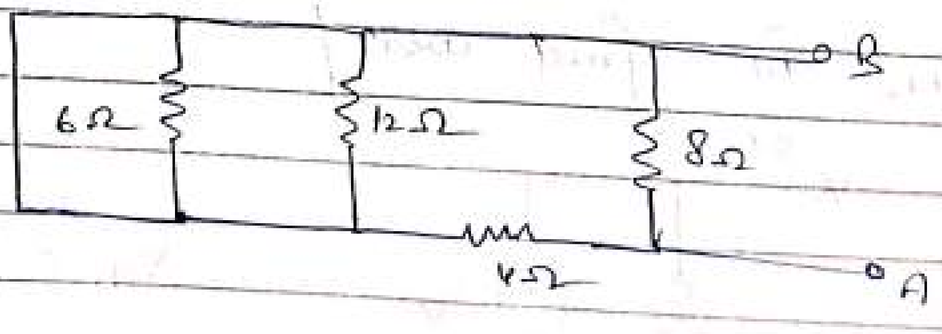
Step-I:-

$$V_{12\Omega} = 48 \times \frac{12}{18} = 32V.$$

$$V_{8\Omega} = 32 \times \frac{8}{12} = 64/3\ V$$

$$V_{4\Omega} = \cancel{32\times 4/12}\ 48 - \frac{64}{3} = \frac{80}{3}\ V.$$

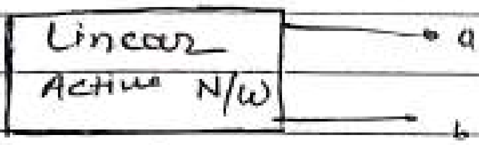
Step-II:-



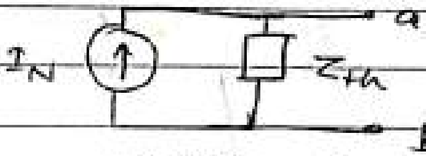
$$R_{th} = \frac{8\Omega}{3}$$

III. Norton Theorem :—

इस Theorem के अनुसार voltage source, current source एवं Impedance से बना किसी Linear circuit का किन्हीं दो Terminal के बीच का व्यवहार एक current source I_N और एक Impedance को Parallel में लगा कर दिखाया जाता है।



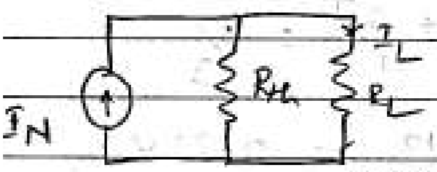
General N/W



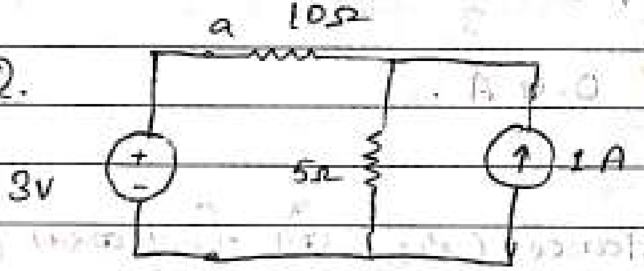
Norton equivalent.

- Norton equivalent और Thevenin equivalent एक दूसरे के Dual हैं।

- Load resistor के द्वारा Norton theorem की मदद से current . $I_L = \frac{R_{th}}{R_{th}+R_L} \times I_N$.

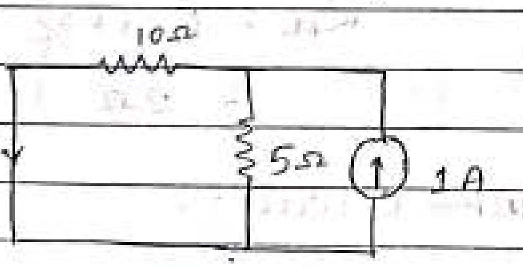


Q.



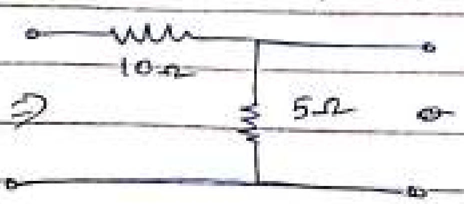
find Norton's equivalent to right of ab terminal.

Step-I :- I_N को find करना।



$I_N = 1 \times \frac{5}{15} = 1/3\ A.$

Step-II :- R_{th} को find करना।



$R_{th} = 15\Omega$

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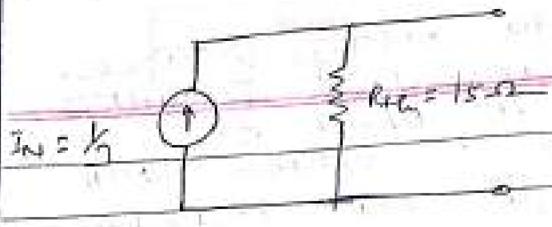
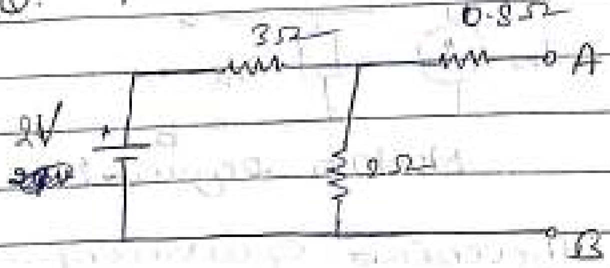
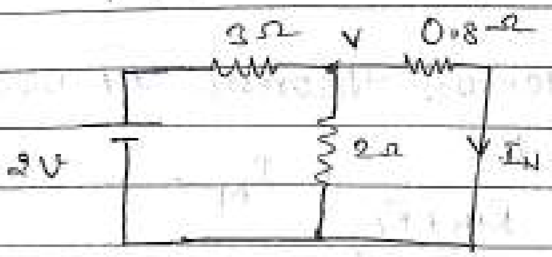
Step-3 :- Norton equivalent circuit :-



Q. find Norton equivalent b/w A & B.



Step-I :- Norton Current (I_N) को find करना है.



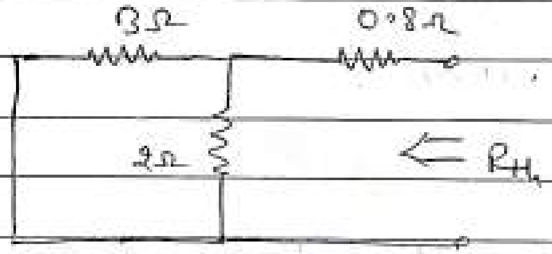
$$\frac{V-2}{3} + \frac{V}{2} + \frac{V}{0.8} = 0.$$

$$V\left[\frac{1}{3} + \frac{1}{2} + \frac{5}{4}\right] = \frac{2}{3}$$

$$V = \frac{2}{3} \times \frac{12}{25} = 0.32\,V.$$

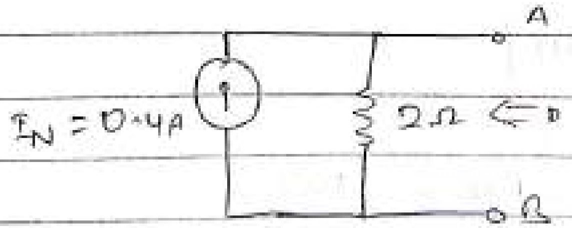
$$I_N = \frac{0.32}{0.80} = 0.4\,A.$$

Step-II :- Thevenin resistance (R_{th}) को find करना



$$R_{th} = 0.8 + 6/5$$
$$= 2\,\Omega$$

Step-III: Norton equivalent Circuit :-



IV. Maximum Power Transfer Theorem :—

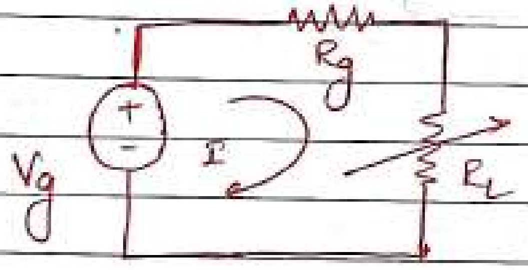
इस Theorem के अनुसार "यदि Source के Impedance Constant हो और Load का Impedance बदलने की स्वतंत्रता हो तो स्त्रोत से लोड को अधिकतम Power उस दशा में Transfer होगी जब Load Impedance Source के Impedance के complex Conjugate के बराबर हो।"

$$\boxed{Z_L = Z_S^*}$$

- यह एक बहुत ही important एवं सबसे ज्यादा Useful theorem है जिसका Use Loud speaker, headphones में किया जाता है।

इस Theorem के अनुसार " ~~Maximum~~ Power Source से Load को maximum transfer होगी जब Load resistance और पुरी Circuit का Thevenin resistance बराबर होगा।"

- जब Source के पास कुछ Internal resistance और load Purely resistive होता है।



एक Voltage Source जिसका Internal resistance R_g और Variable load resistance R_L दिए गए Circuit में दिखाया गया है।

Power in load $R_L = I^2 R_L$

$$P = \left(\frac{V_g}{R_L + R_g}\right)^2 \cdot R_L$$

- Max^m Power को प्राप्त करने के लिए $\frac{dP}{dR_L} = 0$ होना चाहिए।

$$P = \frac{V_g^2}{\frac{1}{R_L}\left(R_L^2 + R_g^2 + 2R_g R_L\right)}$$

$$= \frac{V_g^2}{R_L + \frac{R_g^2}{R_L} + 2R_g}$$

Power max^m होगा जब Denominator minimum होगा।

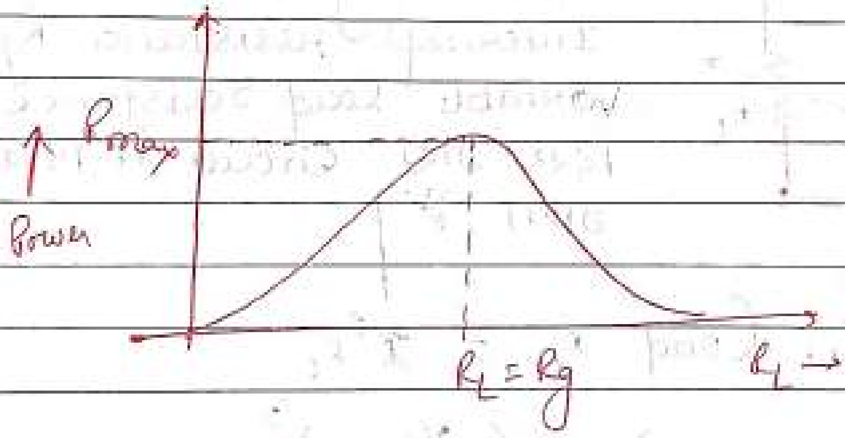
$$D = R_L + \frac{R_g^2}{R_L} + 2R_g$$

$$\frac{dD}{dR_L} = 0 \Rightarrow 1 - \frac{R_g^2}{R_L^2}$$

$$\boxed{R_L = R_g}$$

अतः max^m Power transfer होने के लिए load resistance और source resistance का बराबर होना आवश्यक है।

$$\boxed{\therefore P_{max} = \frac{V_{th}^2}{4R_L}}$$



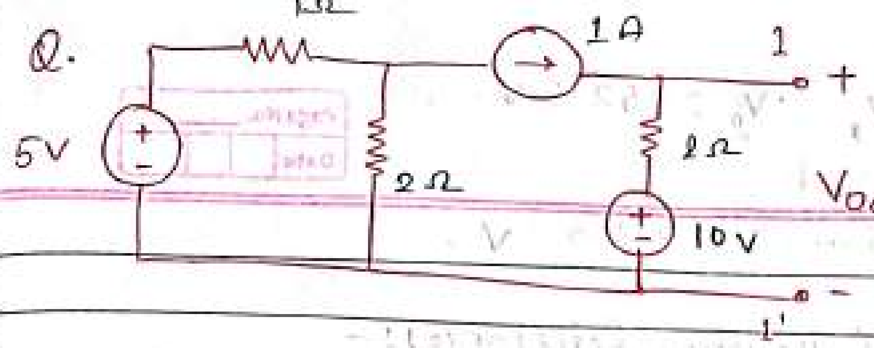
Power variation with load resistance

$$\text{Efficiency} = \frac{\text{Power absorb by load}}{\text{Power available in source}}$$

$$= \frac{I^2 R_L}{I^2 (R_L + R_s)} = \frac{R_L}{2R_L} \times 100$$

$$= 50\%$$

Q.

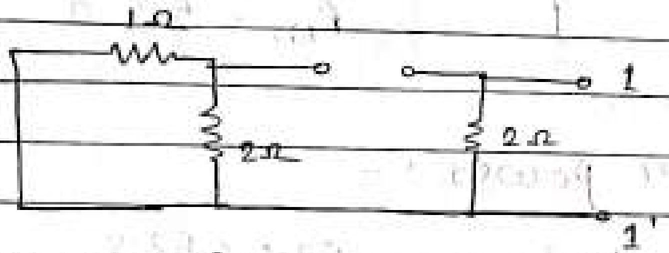


11' के Across किस Ω का Resistance Connect करें जिससे max^m Power transfer हो। find max^m Power ?

Step-I :- Open circuit (V_{oc}) को ज्ञात करें।

$V_{oc} = 2 \times 1 + 10 = 12V.$

Step-II :- Thevenin Resistance (R_{th}) को find करें।



$R_{th} = 2\Omega$.

अतः max^m power transfer theorem के अनुसार,

$R_L = R_{th}$.

$\therefore \boxed{R_L = 2\Omega}$ Ans

Step-III :- max^m power को ज्ञात करना।

$$P_{max} = \frac{V_{th}^2}{4R_L} = \frac{12 \times 12}{4 \times 2} = 18W$$ Ans

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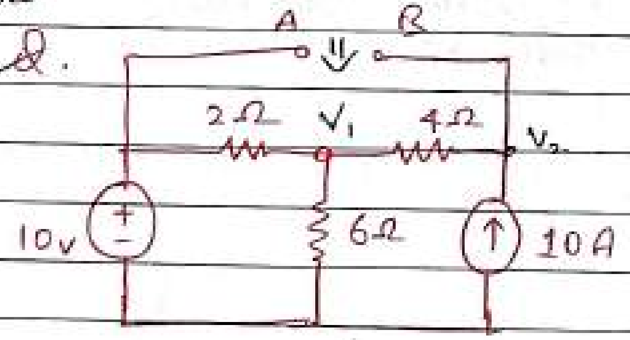
Q.



find R_L so that it will receive the maximum Power. Also determine the value of max^m Power.

Step-I :- Open circuit Voltage (V_{oc}) :-

$$\frac{V_1 - 10}{2} + \frac{V_1}{6} + \frac{V_1 - V_2}{4} = 0.$$

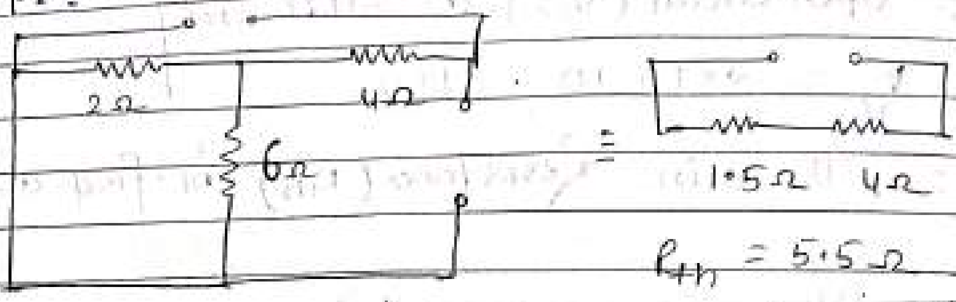
$$\frac{6V_1 - 60 + 2V_1 + 3V_1 - 3V_2}{12} = 0. \Rightarrow 11V_1 - 3V_2 = 60 \quad \text{(I)}$$

$$\frac{V_2 - V_1}{4} = 10 \Rightarrow -V_1 + V_2 = 40 \quad \text{(II)}$$

$\therefore V_1 = 22.5\,V, \quad V_2 = 62.5\,V.$

$V_{th} = 62.5 - 10 = 52.5\ V.$

Step-II :- R_{th} (Thevenin Resistance):-



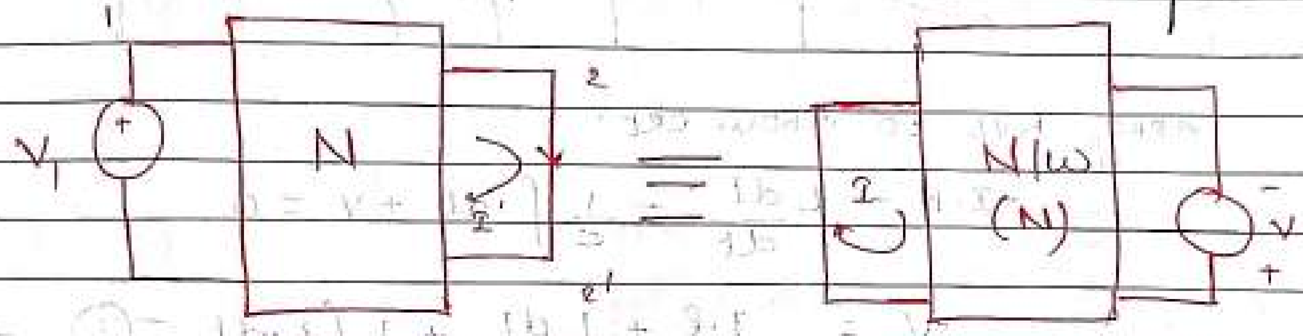
$R_{th} = 5.5\,\Omega$

Step-III: Maxm power :-

$$P_{max} = \frac{V_{th}^2}{4R_L} = \frac{52.5 \times 52.5}{4 \times 5.5} = 125.2\ W$$

V. Reciprocity Theorem:-

इस Theorem के अनुसार "~~response~~ single source network में response और excitation का ratio constant होता है"। इसमें यदि excitation voltage source होगा तो response current source और यदि excitation current source होगा तो ~~excitation~~ response voltage source होगा।



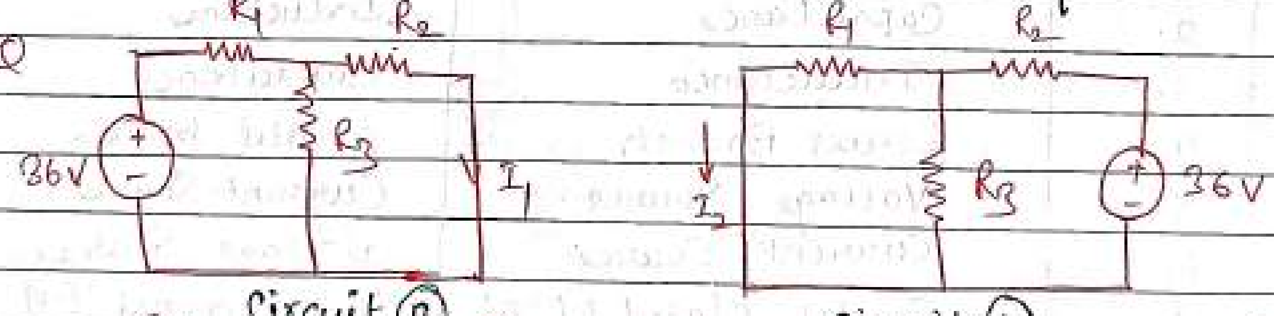
N/w with voltage source V_1

N/w N' with voltage source और response (current) का place interchanged

$$\boxed{\therefore \frac{\text{Response}}{\text{Excitation}} = \text{Constant}}$$

- Reciprocity theorem Passive, linear और bilateral N/w के लिए valid होता है।
- Active N/w के लिए उपयोगी नहीं है।

Q



Circuit (a) Circuit (b)

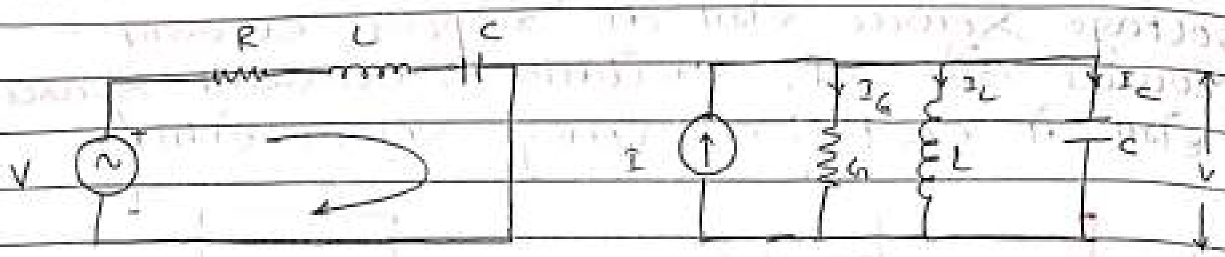
यदि Circuit (a) में $I_1 = 20mA$ है तो Circuit (B) में I_2 क्या होगा यदि दोनों circuit reciprocal है।

Solⁿ:- यदि circuit reciprocal है तो

$$\frac{V_1}{I_1} = \frac{V_2}{I_2} \Rightarrow \frac{36}{20\times10^{-3}} = \frac{36}{I_2}$$

$$\boxed{I_2 = 20mA}$$

VI Duality :- " दो Network एक दूसरे का Dual होंगे यदि दिए गए network का mesh equation और दूसरे network को node equation के बराबर होगा "



Apply KVL to above ckt.

$$-I \cdot R - L\frac{dI}{dt} - \frac{1}{C}\int i\,dt + V = 0$$

$$V = I \cdot R + L\frac{dI}{dt} + \frac{1}{C}\int I\,dt \quad \text{--- (1)}$$

Consider R-L-C Parallel circuit ;

Apply KCL to above circuit,

$$I = I_G + I_L + I_C .$$

$$I = G \cdot V + C \cdot \frac{dV}{dt} + \frac{1}{L}\int V\,dt \quad \text{--- (ii)}$$

Eqn (1) और eqn (ii) Similar है, अतः equation (1) में voltage और eqn (ii) का current I equal है।

	Element	Dual Element
1.	Resistance	Conductance
2.	Capacitance	Inductance
3.	Inductance	Capacitance
4.	Series Branch	parallel branch
5.	Voltage Source.	Current Source
6.	Current Source	Voltage Source
7.	Switch Closed (t=0)	Switch opened (t=0)
8.	Charge.	flux linkage
9	Mesh	Node
10.	Link	Twig

Construction of Dual Network using Graphical Method —

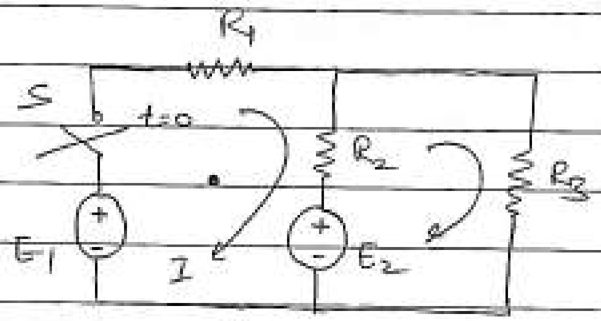
Step-I :- Independent loops को N/w में पहचानो।

Step-II :- हरेक independent loop में non-zero node रखेंगे और Naming करेंगे।

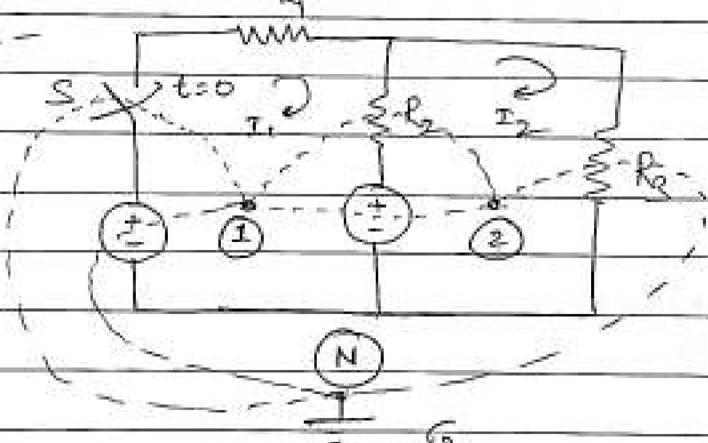
Step-III : Network के बाहर एक zero potential का Node रखेंगे।

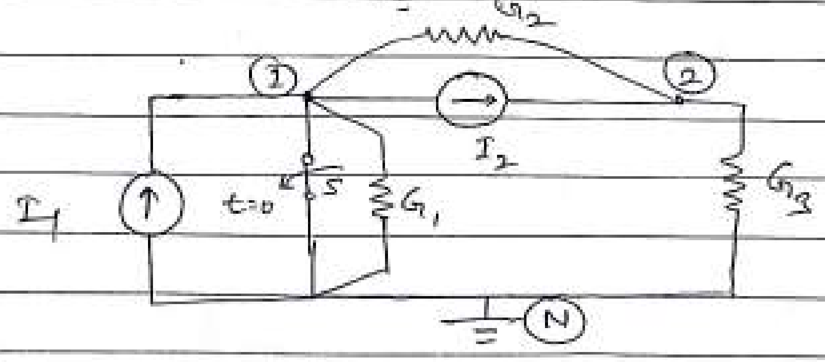
Step-IV :- हरेक element से pass करता हुआ dotted line उस loop के अंदर के Node और zero potential के Node से connect करेंगे।

Step-V :- यदि element दोनों mesh के बिच में connected हो तो Dual में यह element दो Node के बिच में connect होगा।



Dual:-





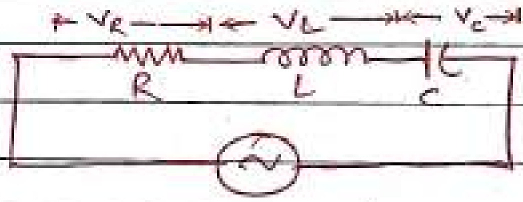
Dual network

Chapter - 2

Resonance.

- Resonance होना circuit series or Parallel a.c circuit में पाया जाता है।
- Definition:- " In an electrical circuit, the condition that exists when the inductive reactance & capacitive reactance are equal and causing electrical energy to oscillate b/w magnetic field of inductor to electric field of capacitor.
- There are two type of Resonance -
  i) Series Resonance.
  ii) Parallel Resonance.

• Series Resonance :-



V volt, f Hz.

$$Z = R + j\omega L + \frac{1}{j\omega C}$$

$$= R + j\omega L - \frac{j}{\omega C}.$$

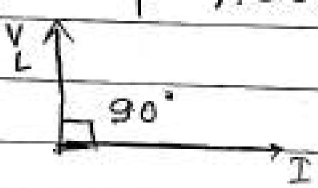
$$Z = R + j(X_L - X_C).$$

where, $X_L = \omega L$, $X_C = \frac{1}{\omega C}$.

Voltage drop Resistance 'R' के Across $(V_R) = IR$.

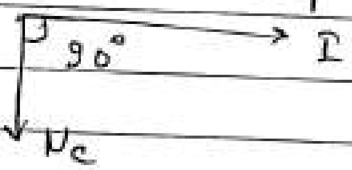
$\longrightarrow I \longrightarrow V_R \Rightarrow$ In-phase

Voltage drop Across inductance 'L' $(V_L) = IX_L$



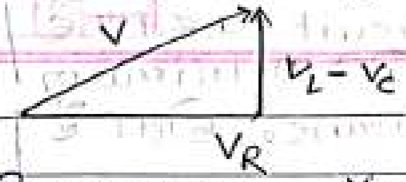
leading I by 90°.

Voltage drop across capacitor C, $V_C = IX_C$



lagging I by 90°

Vector Diagram of RLC series circuit.



At Resonance, $X_L = X_C$

$\therefore Z = R + j0 = R.$

$I_0 = \frac{V}{Z} = \frac{V}{R}$ A.

↳ current at Resonance.

P.F = Power factor $= \cos\phi = \frac{R}{Z} = \frac{R}{R} = 1$

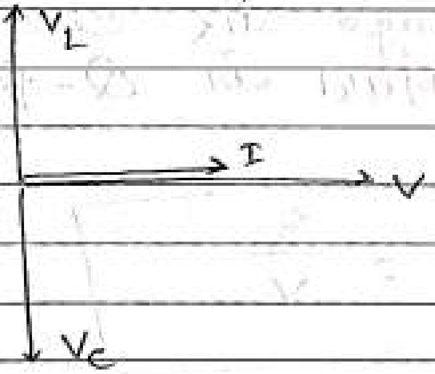
माना कि at resonance f_0 resonance freq. है

$X_L = X_C$

$\omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0^2 = \frac{1}{LC}$

$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow 2\pi f_0 = \frac{1}{\sqrt{LC}}$

$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$ HZ.



- Voltage Vector diagram at resonance ($V_L = V_C$)

RLC Series Resonance circuit की Properties—

i) Power factor Series Resonance circuit का Unity होता है।

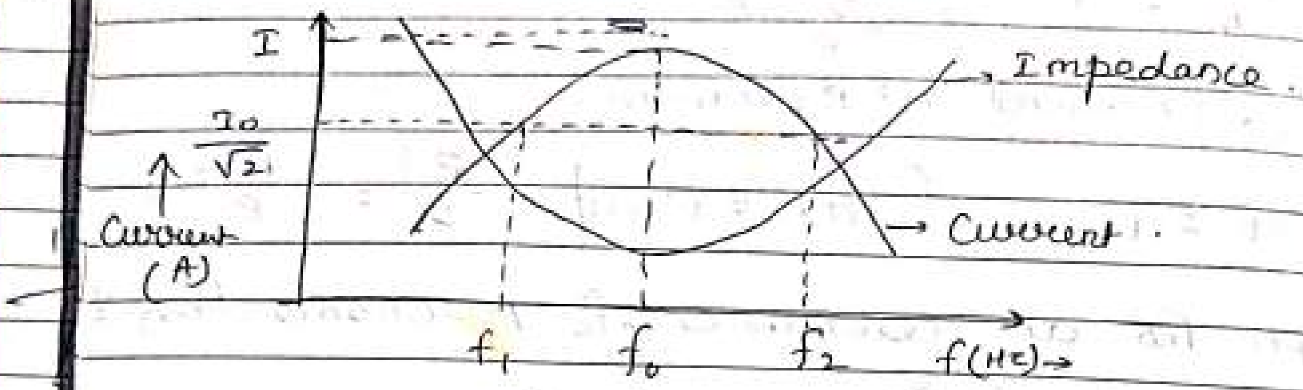
ii) Net reactance zero होता है और Total impedance केवल resistive part से मिलकर बना होता है।

iii) Circuit में current $= \frac{V}{R}$ होता है और यह maximum current होता है।

Current maximum होने के कारण Series RLC Circuit को Acceptor circuit कहते हैं।

(iv) At resonance, circuit में minimum impedance और maximum admittance होता है।

(v) Resonance freq. $f_0 = \frac{1}{2\pi\sqrt{LC}}$ Hz होता है।



Variation of Current & impedance in Series Resonance circuit.

- Q-factor :- (Quality factor) of Series Resonating circuit :-

(a) Circuit में ~~Volt~~ Inductor या capacitor के Across का Voltage और circuit के कुल Voltage के अनुपात को Q-factor कहा जाता है।

$$\boxed{Q = \frac{V_L}{V} = \frac{V_C}{V}}$$

Where, V_L = Voltage Across Inductor
V_C = " " Capacitor
V = Total applied voltage

$$Q = \frac{V_L}{V} = \frac{I_0 X_L}{I_0 R} = \frac{X_L}{R} = \frac{\omega_0 L}{R} \text{ (for inductor)}$$

$$Q = \frac{V_C}{V} = \frac{I X_C}{I_0 R} = \frac{X_C}{R} = \frac{1}{\omega_0 C R} \text{ (for Capacitor)}$$

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\sqrt{LC}} \cdot \frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$\therefore \boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

$$Q = \frac{1}{\omega_0 RC} = \frac{\sqrt{LC}}{RC} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Bandwidth :- Lower और Upper half power frequency band के बीच की frequency को Bandwidth कहते हैं।

$$\omega_2 - \omega_1 = \frac{R}{L}$$

$$\boxed{f_2 - f_1 = \frac{R}{2\pi L}}$$

$$Q = \frac{\omega_0 L}{R} \Rightarrow \frac{Q}{\omega_0} = \frac{L}{R}$$

$$\therefore \frac{Q}{\omega_0} = \frac{1}{\omega_2 - \omega_1} \Rightarrow \boxed{Q = \frac{\omega_0}{\omega_2 - \omega_1}}$$

or, $\boxed{Q = \dfrac{f_0}{f_2 - f_1}} = \dfrac{\text{Resonant freq.}}{\text{Bandwidth}}$

- Selectivity :- It is defined as the ratio of ~~to~~ resonating frequency (f_0) to the bandwidth of the circuit.

$$\boxed{\text{Selectivity} = \frac{f_0}{f_2 - f_1}}$$

- Selectivity $\propto \dfrac{1}{B \cdot W}$

मतलब B.W जितना Narrow होगा Selectivity उतनी ही अच्छी होगी।

Relation b/w Half power frequencies in Series RLC resonating circuit.

$$\omega_1 \omega_2 = \frac{1}{LC} \quad \text{--- (i)}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad \text{--- (ii)}$$

$$\omega_0^2 = \frac{1}{LC}$$

Comparing eqⁿ (i) & (ii) :-

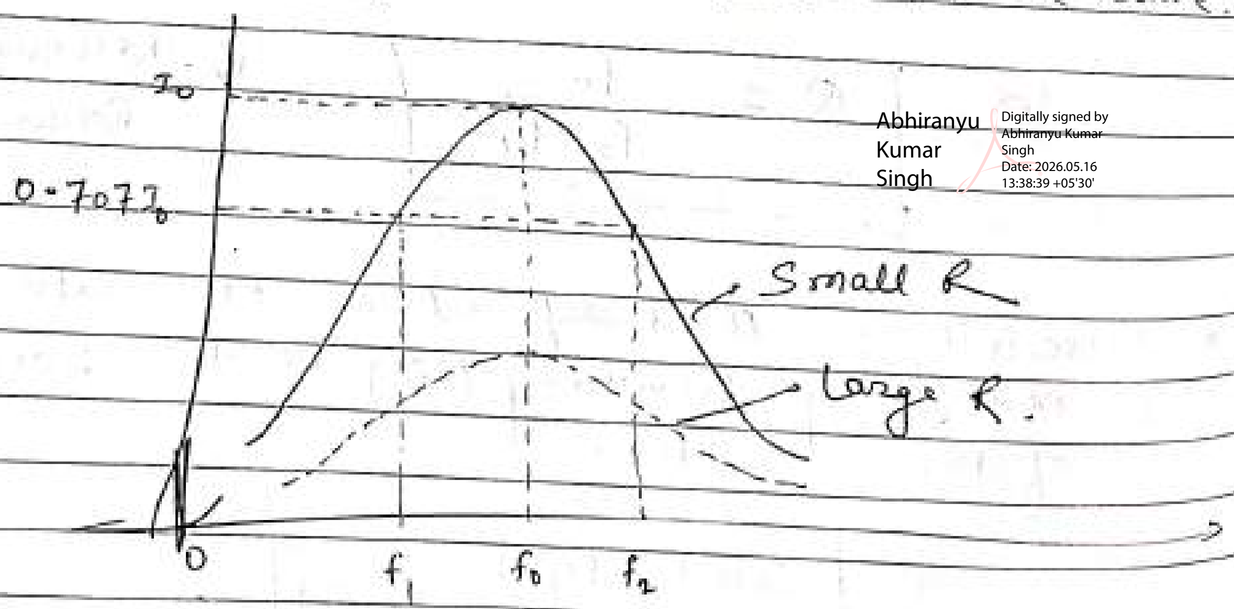
$$\omega_0^2 = \omega_1 \omega_2$$

$$\Rightarrow \omega_0 = \sqrt{\omega_1 \omega_2}$$

or,
$$\boxed{f_0 = \sqrt{f_1 f_2}}$$

i.e. Resonating freq. is geometric mean of the two half power frequencies.

Effect of Resistance on freq. Response Curve.



frequency Response Curve for different R

वैसा circuit जो flat frequency response curve show करता है, ज्यादा responsive और कम selective होता है और flat freq. response large value of R के कारण होता है।

वैसा circuit जो लंबा narrow peak show करता है, less responsive और more selective होता है और लंबा, narrow peak small resistance के कारण होता है।

Q(1) A RLC tank circuit is composed of components having values $R = 0.2\,\Omega$, $L = 100\,mH$, $C = 50\,\mu F$. find the resonant freq. & current at 24V.

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = 71.21\,Hz$$

At resonance, $I = \frac{V}{R} = \frac{24}{0.2} = 120\,A$.

(2) A Series RLC circuit has inductance of 10 mH & resistance of $2\,\Omega$. What is the value of capacitance that will produce resonance. Also find current at resonance freq. & max^m instantaneous energy stored in inductance at resonance. If power supply is 230 V, 10000 HZ.

$C = 0.025\,\mu F$, $I_0 = \frac{230}{2} = 115$, $E = \frac{1}{2} L I_{max}^2$

$= \frac{1}{2} L (\sqrt{2} I_{rms})^2$

$= 132.25\,J$.

(3) What is resonant freq. of a Series RLC circuit where $R = 10\,\Omega$, $L = 25\,mH$, $C = 100\,\mu F$? find Q-factor also.

$f_0 = 100.71\,HZ$, $Q = 1.58$.

(4) Calculate Half power freq. of a series resonant ckt where the resonance freq. is $150 \times 10^3\,HZ$ and B.W is 75 KHZ.

$f_2 - f_1 = B.W$ & $f_r = \sqrt{f_1 f_2}$

$f_2 - f_1 = 75$ & $\sqrt{f_1 f_2} = 150$.

$f_1 = 117$ KHZ, $f_2 = 192$ KHZ

2012

Q5 एक श्रेणी LC परिपथ का $L = 100\,PH$, $C = 2500\,\mu F$, $Q = 70$ मान्य है। ज्ञात कीजिए –

(i) Resonant freq. f_0 (ii) $f_2 - f_1$

$f_0 =$

$f_2 = 191.9$ KHZ, $f_1 = 266.9$ KHZ

2011-12

Q6. A Series RLC circuit has the following Parameters $R = 10\Omega$, $L = 0.01H$, $C = 100\mu F$.

(i) find resonant frequency, $f_0 = 159.154$ HZ

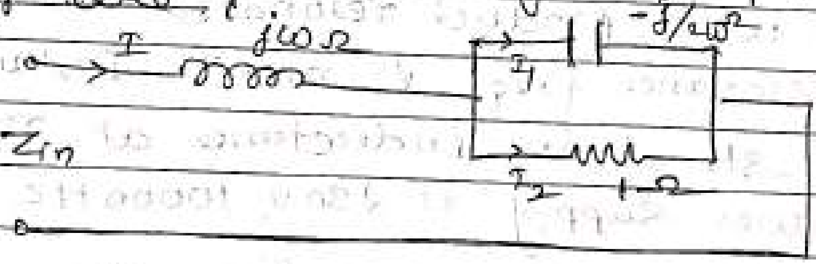
(ii) Calculate Quality factor $Q = 1$

(iii) find Bandwidth. $B.W = 159.154$ HZ

(iv) find upper & lower power frequency.

$f_2 = 257.512$ HZ, $f_1 = 98.36$ HZ

(7) Find the frequency at which the given circuit will be at resonance. ~~If the Capacitor and inductor are~~ i.



At resonance, Imaginary part of input impedance must be zero.

$$Z_{in} = j\omega + \frac{(-j/2\omega) \times 1}{-j/2\omega + 1}$$

$$Z_{in} = j\omega - \frac{j}{2\omega - j}$$

$$= j\omega - \frac{j(2\omega + j)}{4\omega^2 + 1}$$

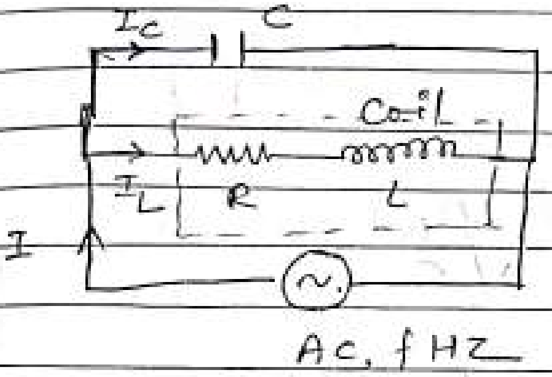
Imaginary part $= j\omega - \frac{j \times 2\omega}{1 + 4\omega^2} = 0$

$$\omega = \frac{2\omega}{1+4\omega^2}$$

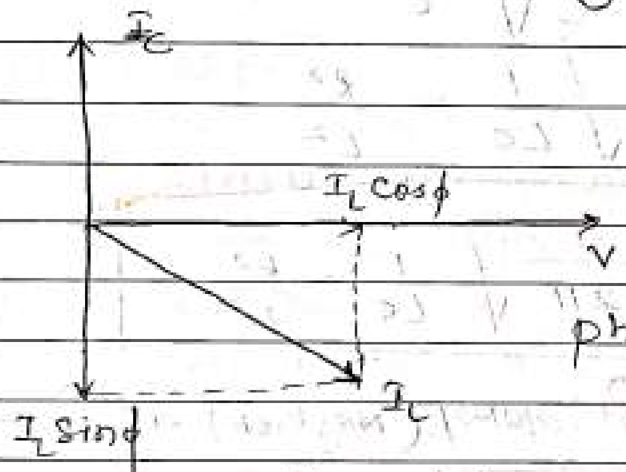
$$1+4\omega^2 = 2 \Rightarrow 4\omega^2 = 1$$

$$\omega = \frac{1}{2} = 0.5 \ rad/sec$$

- Parallel Resonance :-



दिया गया diagram parallel resonating circuit की तरह काम करेगा जहाँ Capacitance(C) के Parallel में Resistance और inductance as series combination connected है। इस पूरे combination को A.C voltage source और variable frequency से connect करते हैं।



phasor diagram of parallel A.C circuit

$$I_C = \frac{V}{X_C}$$

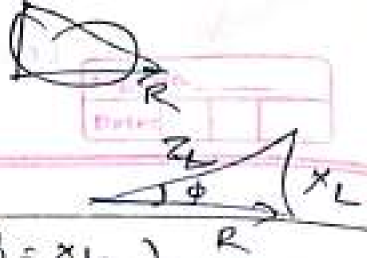
and, $$I_L = \frac{V}{Z_L} = \frac{V}{\sqrt{R^2+X_L^2}} = \frac{V}{\sqrt{R^2+(\omega L)^2}}$$

$$\cos\phi = \frac{R}{Z}.$$

At resonance, Capacitive current (I_C) और Inductive current (I_L) दोनो बराबर होंगे।

$$I_C = I_L \sin\phi.$$

Or, $\frac{V}{X_C} = \frac{V}{Z_L} \times \frac{X_L}{Z_L}$.



$\left(\text{Where } \sin\phi = \frac{X_L}{Z_L} \right)$

or, $Z_L^2 = \frac{1}{\omega_0 C} \times \omega_0 \times L = \frac{L}{C}$.

or, $Z_L = \sqrt{\frac{L}{C}}$ —— (1)

or, $\sqrt{R^2 + \omega_0^2 L^2} = \sqrt{\frac{L}{C}}$

$\omega_0^2 L^2 = \frac{L}{C} - R^2$

$\omega_0 = \frac{1}{L}\sqrt{\frac{L}{C} - R^2}$

$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$

$$\boxed{f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$

If Coil का resistance को नगण्य (Neglect) कर दें -

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$$

f_0 = resonant frequency

Resonance पर reactive component में बहने वाले Current I_C और I_L sinϕ दोनों Balance हो जाते हैं और $I_L \cos\phi$ वह Current कहलाएँ

$I = I_L \cos\phi$

$\frac{V}{Z_\Omega} = \frac{V}{Z_L} \times \frac{R}{Z_L}$

Z_Ω = Total impedance of Parallel circuit

$$Z_\Omega = \frac{Z_L^2}{R} = \frac{L/C}{R} \quad \left(\text{from eq}^n \text{ (1)}\right)$$

$$\boxed{Z_\Omega = \frac{L}{CR}}$$

R की Value बहुत कम होती है, इसलिए Total Circuit का Impedance बहुत ज्यादा होता है और current बहुत कम होता है। इसलिए इसे Rejector Circuit कहते हैं।

Resonance पर current, $= \frac{V}{Z_\Omega} = \frac{V}{L/CR}$

$$= \frac{V \times CR}{L}$$

Power factor, $\cos\phi = 1$

Properties :—

(I) Net impedance resonance पर Parallel circuit का maximum और L/CR के बराबर होता है।

(II) Resonance पर Parallel circuit का current minimum और $V \times CR/L$ के बराबर होता है।

(III) Admittance minimum होता है।

(IV) Power factor unity होता है।

(V) Resonant frequency Parallel circuit के लिए,

$$\left[f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}\right]$$

Q - factor of Parallel Resonating Circuit :—

यह Capacitance में बहने वाले current और Parallel circuit में बहने वाले current के ratio को Q - factor कहते हैं।

$$Q = \frac{I_C}{I} = \frac{V/X_C}{V/Z_R} = \frac{Z_R}{X_C}$$

$$= \frac{L}{CR} \times \frac{1}{1/\omega_0 C} = \frac{L}{\cancel{C}R} \cdot \omega_0 \cancel{C}$$

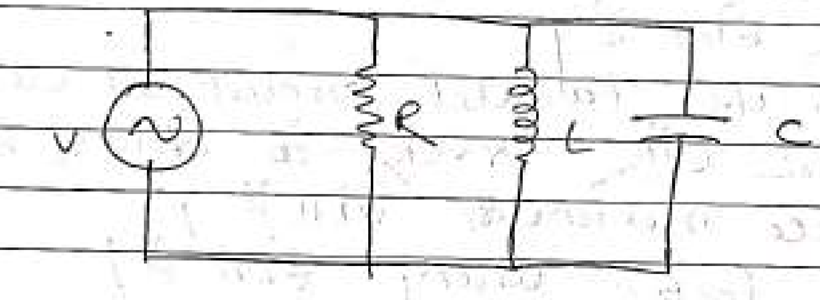
$$\boxed{Q = \frac{L}{R} \times \frac{1}{\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

Parallel resonating circuit को rejector circuit या anti-resonating circuit भी कहते है।

$$\boxed{\text{Bandwidth} = f_2 - f_1 = \frac{f_0}{Q}}$$

$$\boxed{\text{Selectivity} = \frac{f_0}{f_2 - f_1} = Q_0}$$

(2)



Admittance, $Y = \frac{1}{R} + j\omega C + \frac{1}{j\omega L}$

$$Y = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

Frequency जिसपर circuit resonance करेगा.

$$\omega_0 C - \frac{1}{\omega_0 L} = 0.$$

$$\omega_0 C = \frac{1}{\omega_0 L}$$

$$\omega_0^2 = \frac{1}{LC} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\boxed{\therefore f_0 = \frac{1}{2\pi\sqrt{LC}}}$$

$$\boxed{\text{Bandwidth} = \omega_2 - \omega_1 = \frac{1}{RC}} \boxed{= f_2 - f_1}$$

$$Q = \frac{f_0}{f_2 - f_1} = \frac{f_0}{1/RC}$$

$$\boxed{Q = f_0 \times RC}$$

Q A series RLC circuit has $R = 2\Omega$, $L = 2mH$, $C = 10\mu F$. Calculate (i) Q-factor of circuit (ii) Bandwidth (iii) Resonant frequency (iv) Half power freq. f_1 & f_2

1125.39

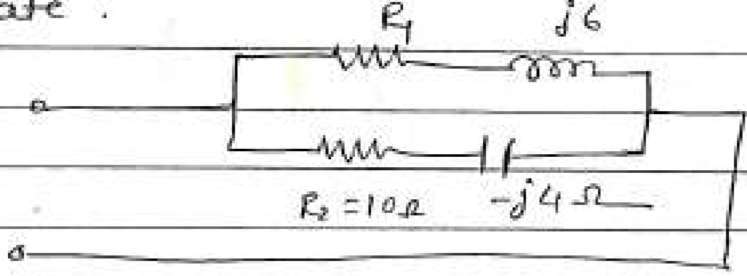
(iii) $f_0 = 1126.13 \, HZ$ (i) $Q = 7.07$.

(ii) $B \cdot W = f_2 - f_1 = \frac{R}{2\pi L} = 159.23 \, HZ$

(iv) $f_1 = 1049.16 \, HZ$

$f_2 = 1208.4 \, HZ$.

Q find the value of R_1 such that circuit will resonate.



Equivalent admittance,

$$Y = \frac{1}{R_1 + j6} + \frac{1}{10 - 4j}$$

$$= \frac{R_1 - 6j}{R_1^2 + 36} + \frac{10 + 4j}{100 + 16}.$$

$$= \frac{R_1}{R_1^2 - 36} - \frac{6j}{R_1^2 - 36} + \frac{10}{116} + \frac{4j}{116}$$

$$= \frac{R_1}{R_1^2-36} + \frac{10}{\cancel{84}\,116} + j\left(\frac{4}{\cancel{84}\,116} - \frac{6}{R_1^2-36}\right)$$

At Resonance, imaginary part = 0.

$$\frac{\cancel{4}^{2}}{\cancel{84}_{42}} = \frac{\cancel{6}^{3}}{R_1^2-36}.$$

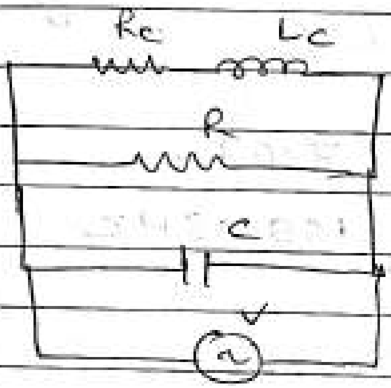
$$\cancel{R_1^2 - 36 = 126}$$

$$\cancel{R_1^2 = 162}$$

$$\cancel{R_1 = \sqrt{162}}$$

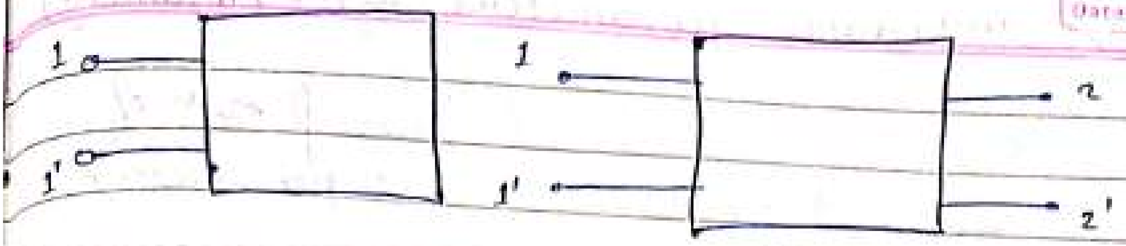
$$R_1 = \sqrt{138}$$

(Q) Find resonant frequency of given circuit.



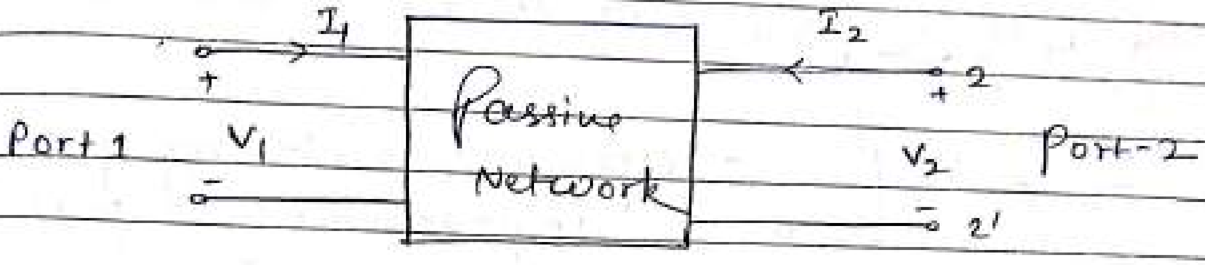
$$Y = \frac{1}{R} + j\omega C + \frac{1}{R_c + j\omega L_c}$$

~~Network filters,~~ Two port Network



One port N/W   Two port Network

* Two port Network :-



Name of Parameter	Independent Variables	dependent Variables	Defining equation
(i) Open circuit impedance	I_1, I_2	V_1, V_2	$V_1 = Z_{11}I_1 + Z_{12}I_2$ $V_2 = Z_{21}I_1 + Z_{22}I_2$
(ii) Short circuit admittance	V_1, V_2	I_1, I_2	$I_1 = V_1 y_{11} + V_2 y_{12}$ $I_2 = V_1 y_{21} + V_2 y_{22}$
(iii) Transmission parameter	V_2, I_2	V_1, I_1	$V_1 = AV_2 - BI_2$ $I_1 = CV_2 - DI_2$
(iv) Hybrid	I_1, V_2	V_1, I_2	$V_1 = h_{11}I_1 + h_{12}V_2$ $I_2 = h_{21}I_1 + h_{22}V_2$
(v) Inverse Transmission	V_1, I_1	V_2, I_2	$V_2 = AV_1 - BI_1$ $I_2 = CV_1 - DI_1$
(vi) Inverse Hybrid	I_2, V_1	V_2, I_1	$I_1 = g_{11}V_1 + g_{12}I_2$ $V_2 = g_{21}V_1 + g_{22}I_2$

1) Open circuit Impedance Parameters : (Z-Parameter)

$V_1 = Z_{11} I_1 + Z_{12} I_2$ (General eq^n of

$V_2 = Z_{21} I_1 + Z_{22} I_2$ Z-parameter)

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

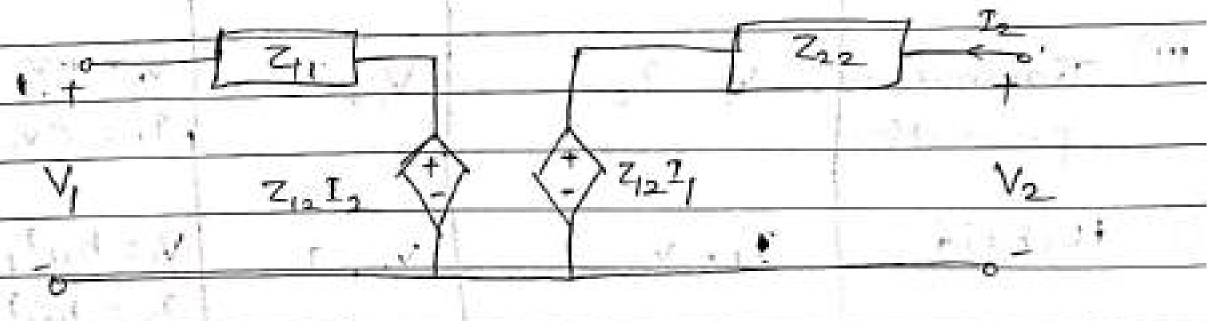
यदि I_1 या I_2 zero होगा तो 4 different parameter हमे मिलेंगे।

$Z_{11} = \left.\dfrac{V_1}{I_1}\right|_{I_2=0}$ = open circuit driving point impedance of port 1.

$Z_{21} = \left.\dfrac{V_2}{I_1}\right|_{I_2=0}$ open circuit forward transfer impedance.

$Z_{12} = \left.\dfrac{V_1}{I_2}\right|_{I_1=0}$ Open circuit reverse transfer impedance

$Z_{22} = \left.\dfrac{V_2}{I_2}\right|_{I_1=0}$ open circuit driving point impedance of port 2.

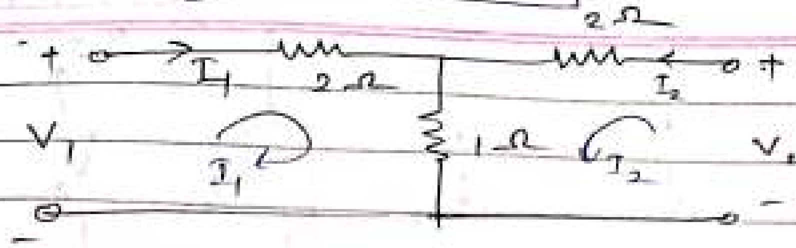


Two Source equivalent circuit of Z-parameter

If Network is reciprocal, $Z_{21} = Z_{12}$

Symmetrical Network मतलब Circuit को 2 Part में बांट सकते हैं और दोनों Part mirror image की तरह काम करता है।

$$\boxed{Z_{22} = Z_{11}}$$

① 

$$\left.\begin{aligned} V_1 &= 3I_1 + I_2 \\ V_2 &= +I_1 + 3I_2 \end{aligned}\right\}$$

Comparing with general equation of Z-parameter

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$\therefore Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 3 & +1 \\ +1 & 3 \end{bmatrix}$$

② Short circuit Admittance Parameter (Y-Parameter)

Y-parameter को इन दो equation से define करते हैं।

$$I_1 = y_{11} V_1 + y_{12} V_2$$

$$I_2 = y_{21} V_1 + y_{22} V_2$$

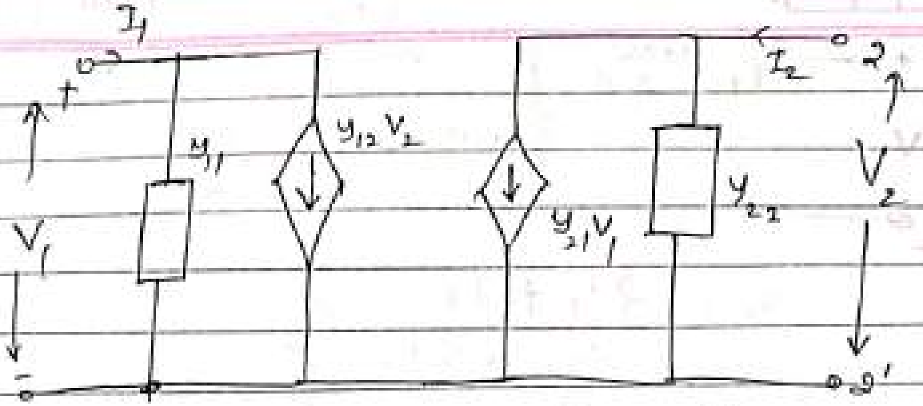
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$y_{11} = \left.\dfrac{I_1}{V_1}\right|_{V_2=0}$ Short circuit driving point admittance of port 1.

$y_{21} = \left.\dfrac{I_2}{V_1}\right|_{V_2=0}$ Short circuit forward transfer admittance.

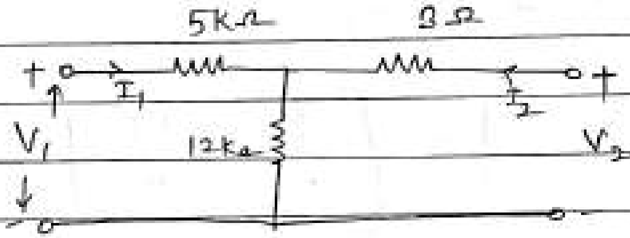
$y_{12} = \left.\dfrac{I_2}{V_2}\right|_{V_1=0}$ Short circuit reverse transfer admittance

$y_{22} = I_2/V_2 \,\big|_{V_1=0}$ Short circuit driving point admittance of port 2



यदि Network reciprocal होगा, तो $\boxed{y_{12} = y_{21}}$

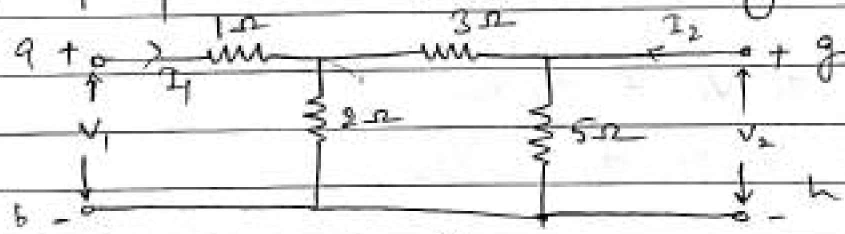
यदि " Symmetrical होगा तो $\boxed{y_{11} = y_{22}}$

Q. 

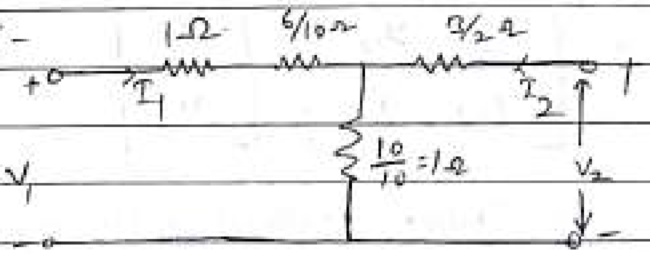
find Z-parameter of circuit.

$$Z_{11} = 17\times10^3\,\Omega, \quad Z_{12} = 12\times10^3\,\Omega$$
$$Z_{21} = 12\times10^3\,\Omega, \quad Z_{22} = 15\times10^3\,\Omega$$

Q. find the Z-parameter of circuit.



Method - I :-



$$\therefore V_1 = (1\cdot6+1)I_1 + I_2$$
$$V_1 = 2\cdot6\,I_1 + I_2 \quad \text{—— (1)}$$

$$V_2 = 1\,I_1 + 2\cdot5\,I_2 \quad \text{—— (11)}$$

$$\therefore Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 2\cdot6 & 1 \\ 1 & 2\cdot5 \end{bmatrix}$$

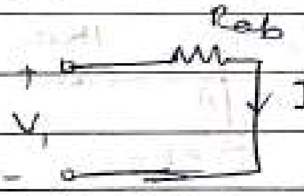
∵ $Z_{12} = Z_{21} = 1\,\Omega$, Network reciprocal है।

Method - 2 :-

माना कि current $I_2 = 0$, मतलब g h terminal open circuited है।

$$\therefore R_{ab} = \cancel{3\|}\left[(3+5)\,\|\,2\right] + 1$$

$$R_{ab} = 1.6 + 1 = 2.6\,\Omega$$



$$\therefore V_1 = R_{ab} \cdot I_1$$

$$\boxed{Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 2.6\,\Omega}$$

माना कि $5\,\Omega$ में बहने वाला current I_x है तो

$$I_x = I_1 \times \frac{2}{8+2}$$

$$I_x = \frac{I_1}{5}$$

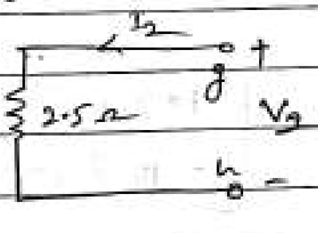
$$\therefore V_2 = I_x \times 5 = \frac{I_1}{\cancel{5}} \times \cancel{5}$$

$$\boxed{Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = 1\,\Omega}$$

Now, Current $I_1 = 0$ मान लिया और ab terminal open circuit कर दिया।

$$R_{gh} = \left[(3+2)\,\|\,5\right] = \frac{25}{10} = 2.5\,\Omega$$

∴



$$V_2 = 2.5 \times I_2$$

$$\boxed{\left.\frac{V_2}{I_2}\right|_{I_1=0} = Z_{22} = 2.5\,\Omega}$$

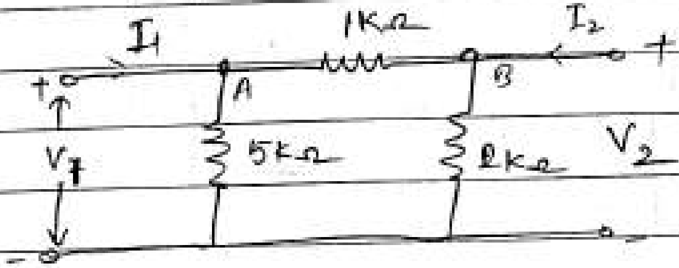
माना $2\,\Omega$ में बहने वाला current I_y है।

$$\therefore I_y = I_2 \times \frac{5}{10} = \frac{I_2}{2}$$

$\therefore V_1 = \cancel{\frac{I_2}{2}} I_y \times 2 = \frac{I_2}{\cancel{2}} \times \cancel{2}$

$$\boxed{\left.\frac{V_1}{I_2}\right|_{I_1=0} = Z_{12} = 1\,\Omega}$$

Q. एक π-network show किया गया है। इसका Y-parameter ज्ञात करें।



KCL at Node A,

$$I_1 = \frac{V_1}{5k\Omega} + \frac{V_1 - V_2}{1k\Omega}$$

$$I_1 = \frac{6}{5k\Omega}V_1 - \frac{V_2}{1k\Omega} \quad \text{— (I)}$$

KCL at node B,

$$I_2 = \frac{V_2}{2k\Omega} + \frac{V_2 - V_1}{1k\Omega}$$

$$I_2 = -\frac{V_1}{1k\Omega} + \frac{3}{2k\Omega}V_2 \quad \text{— (II)}$$

$\therefore$ Comparing eqⁿ (I) & (II) with general equation of ~~Z~~ Y-parameter.

$$\therefore I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

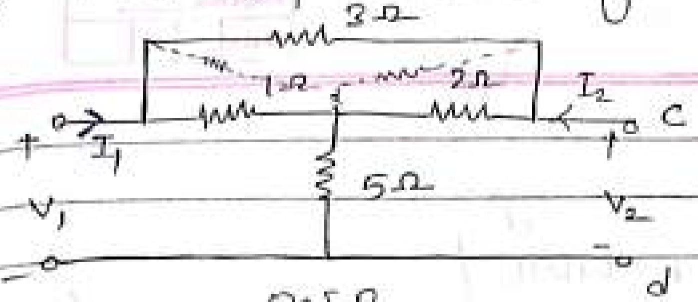
mho

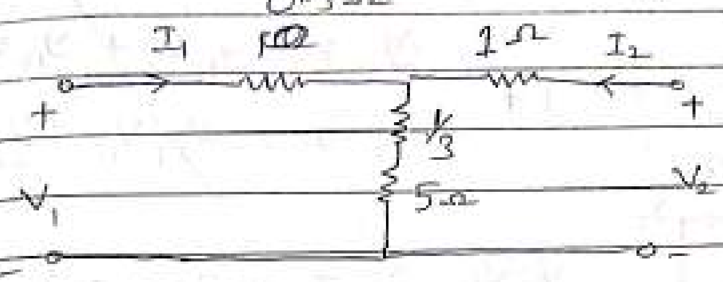
$$\therefore y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} 1.2\times10^{-3}\,\mho & -10^{-3}\,\mho \\ -10^{-3}\,\mho & 1.5\times10^{-3}\,\mho \end{bmatrix}$$

[Unit of admittance = mho (℧)]

April-2016

Q Find Z-parameter of circuit.





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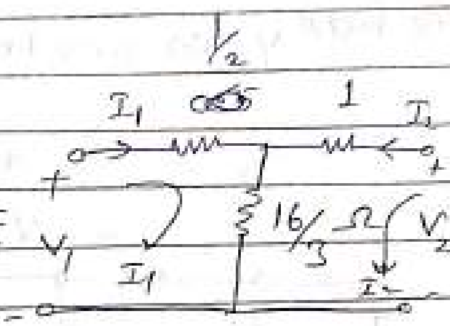


$$\cancel{V_1 = 0.5 I_1 + (I_1+I_2) \times 5.33}.$$

$$\cancel{= 5.88 I_1 + 5.33 I_2}$$

$$V_1 = \frac{1}{2} I_1 + \left(\frac{16}{3}\right)(I_1+I_2)$$

$\frac{3+32}{6}$

$$= I_1\left(\frac{1}{2} + \frac{16}{3}\right) + \frac{16}{3} I_2$$

$$= \frac{35}{6} I_1 + \frac{16}{3} I_2 \quad \text{— (I)}$$

$$V_2 = I_2 + 16/3\,(I_2 + I_1).$$

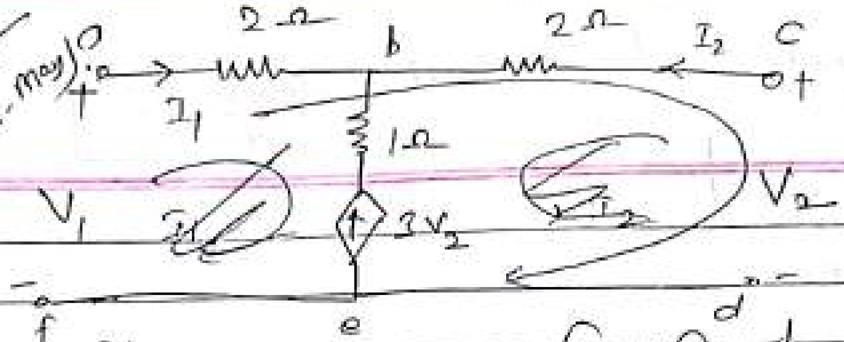
$$= \frac{16}{3} I_1 + \frac{19}{3} I_2 \quad \text{— (II)}$$

Comparing eq^n (I) & (II) with general eq^n of z-parameter.

$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 35/6\,\Omega & 16/3\,\Omega \\ 16/3\,\Omega & 19/3\,\Omega \end{bmatrix}$$

A

Q. (2016. may)

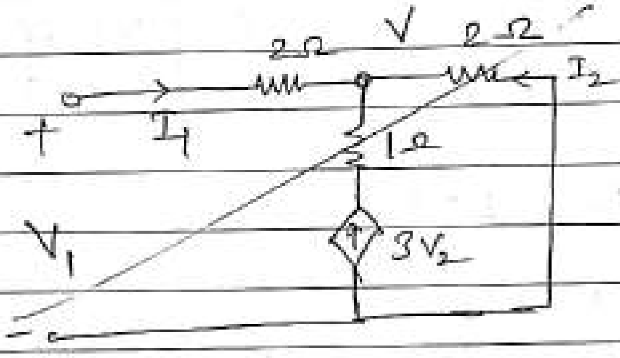


दिए चित्र में Y-parameter निकालिए।

$Q_1\ I_1 = y_{11}V_1 + y_{12}V_2$

$I_2 = y_{21}V_1 + y_{22}V_2$

~~$V_1 = 2I_1 + 1(I_1 + I_2) +$~~



$\frac{V - V_1}{2} + I_1 + 3V_2 + I_2 = 0$ — (i)

$V = 3V_2$ — (ii)

$\therefore I_1 = \frac{V_1 - V}{2} = \frac{V_1 - 3V_2}{2}$

$\therefore I_1 = \frac{1}{2}V_1 - \frac{3}{2}V_2$ —— 1

Circuit के outer loop abcdef में KVL लगाते हैं,

$2I_1 - 2I_2 + V_2 = V_1$ — (i)

Node (b) पर KCL लगाते हैं,

$I_1 + I_2 + 3V_2 = 0.$ — (ii)

Now, $I_2 = -I_1 - 3V_2$. — (ii)

$I_1 = -I_2 - 3V_2$ — (iii)

$\therefore$ Eq^n (ii) को eq^n (i) पर रखते हैं,

$2I_1 - 2(-I_1 - 3V_2) + V_2 = V_1$

$4I_1 + 6V_2 + V_2 = V_1$

$I_1 = \frac{1}{4}V_1 - \frac{7}{4}V_2$ — (iv)

Eq^n (iii) को eq^n (i) पर रखते हैं,

$2(-I_2 - 3V_2) - 2I_2 + V_2 = V_1$

$-4I_2 - 6V_2 + V_2 = V_1$

$-4I_2 = V_1 + 5V_2$

$$I_2 = -\frac{1}{4}V_1 - \frac{5}{4}V_2 \quad \text{--- (v)}$$

Equation (iv) और (v) को general equation से compare करेंगे,

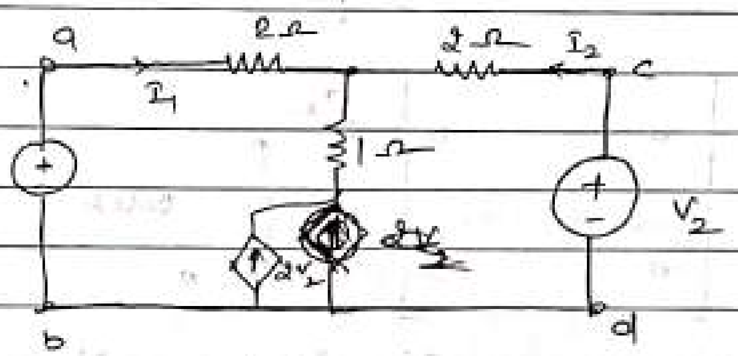
$I_1 = y_{11}V_1 + y_{12}V_2$

$I_2 = y_{21}V_1 + y_{22}V_2$

$\therefore\ y_{11} = \frac{1}{4}\ ℧ \quad , \quad y_{12} = -7/4\ ℧$

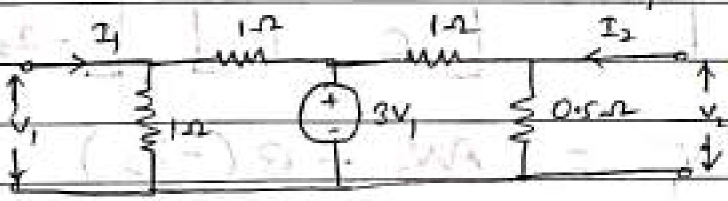
$y_{21} = -\frac{1}{4}\ ℧ \quad , \quad y_{22} = -5/4\ ℧$

Q



Ans:- $y = \begin{bmatrix} 1/4\ \text{mho} & -5/4\ ℧ \\ -1/4\ ℧ & -3/4\ ℧ \end{bmatrix}$

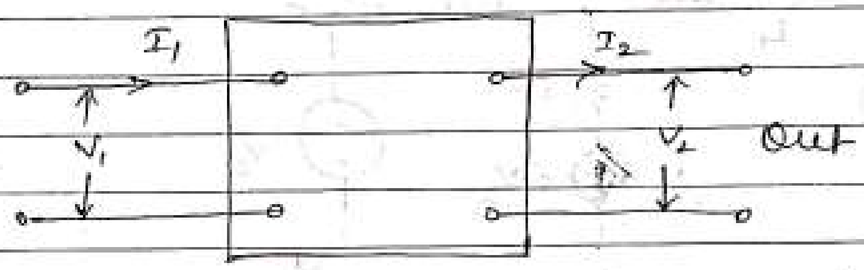
Q. Z-parameter ज्ञात करें।



(3) ABCD Parameter :-

Power transmission engineering के क्षेत्र में ABCD parameter का सबसे ज्यादा उपयोग होता है।

- ABCD parameter को हम Transmission Parameter भी कहते हैं।



ABCD Parameter में input और output port पर voltage और current का Representation.

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$V_1 = AV_2 + B(-I_2)$$

$$I_1 = CV_2 + D(-I_2)$$

$$\boxed{\begin{aligned} \therefore\ V_1 &= AV_2 - BI_2 \quad \text{—— (i)} \\ I_1 &= CV_2 - DI_2 \quad \text{—— (ii)} \end{aligned}}$$

यदि $I_2 = 0$ (Output side को open circuit कर दें)

$A = \left.\frac{V_1}{V_2}\right|_{I_2 = 0}$ (from eq^n 1)

reverse voltage ratio

$C = \left.\frac{I_1}{V_2}\right|_{I_2 = 0}$ (from eq^n 11)

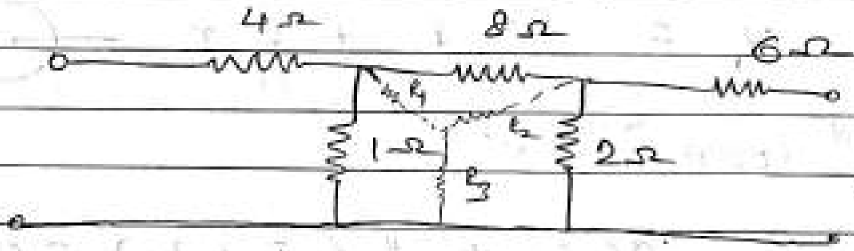
transfer Admittance

यदि Output side को short circuit करें
(i.e $V_2 = 0$)

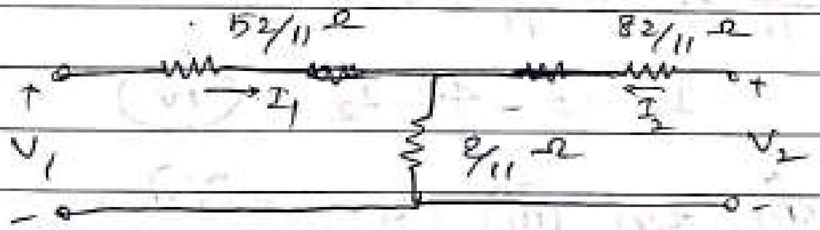
$\therefore$ $B = \left.\frac{V_1}{-I_2}\right|_{V_2 = 0}$ → transfer impedance

$D = \left.\frac{I_1}{-I_2}\right|_{V_2 = 0}$ → reverse current ratio.

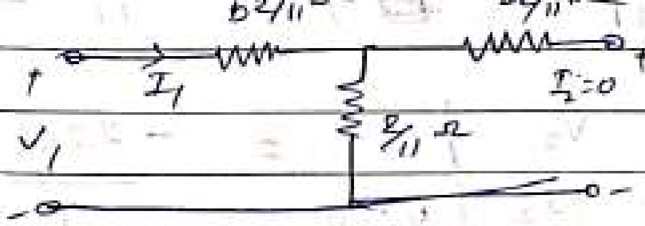
Q. find the transmission Parameter of N/W.



$R_1 = \frac{8}{11}\,\Omega$, $R_2 = \frac{16}{11}\,\Omega$, $R_3 = \frac{2}{11}\,\Omega$



1 open circuit terminal 2 :-



$V_1 = \frac{52}{11} I_1 + \frac{2}{11} I_1$

$V_1 = \frac{54}{11} I_1$ — (i)

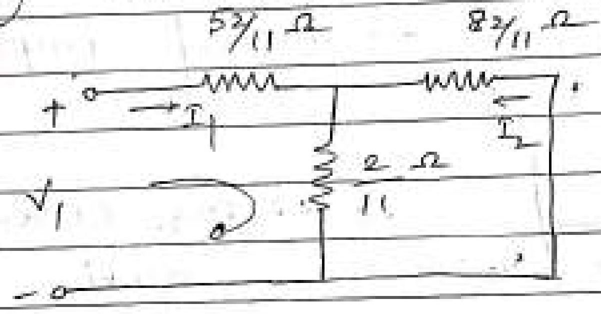
$V_2 = \frac{2}{11} I_1$ — (ii)

$$V_2 = \left(\frac{2}{11}\right)\left(\frac{11}{54}\right)V_1$$

$$\boxed{A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 27}$$

$$\boxed{C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{11}{2} = 5.5 \text{ mho}}$$

(ii) Short circuit the port-2, means $V_2 = 0$



$$V_1 = +\frac{52}{11}I_1 + \frac{2}{11}(I_1 + I_2)$$

$$V_1 = \frac{54}{11}I_1 + \frac{2}{11}I_2 \quad \text{— (iii)}$$

2nd loop में KVL:-

$$\frac{82}{11}I_2 + \frac{2}{11}(I_2 + I_1) = 0$$

$$\frac{2}{11}I_1 + \frac{84}{11}I_2 = 0$$

$$I_1 = -42\, I_2 \quad \text{— (iv)}$$

eqn (iv) को eqn (iii) पर रखेंगे —

$$V_1 = \frac{54}{11} \cdot \left(\frac{-42}{}\right)I_2 + \frac{2}{11}I_2$$

$$B = \left.\frac{V_2}{-I_2}\right|_{V_2=0} = \frac{-2266}{-11} = 206\,\Omega$$

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = \frac{-42}{-1} = 42$$

$$\therefore [T] = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 27 & 206 \\ 5.5 & 42 \end{bmatrix}$$

(4) Hybrid Parameter :- (h-Parameter).

- इसे h-Parameter Representation का उपयोग electronic circuit और Components जैसे की Transistors को बनाने में किया जाता है।
- इसमें दोनों Short circuit और open circuit terminal Condition का उपयोग किया जाता है इसलिए इसे Hybrid Parameter Representation कहते हैं।

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$\therefore \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Short Circuit Condition at port-2 ($V_2 = 0$)

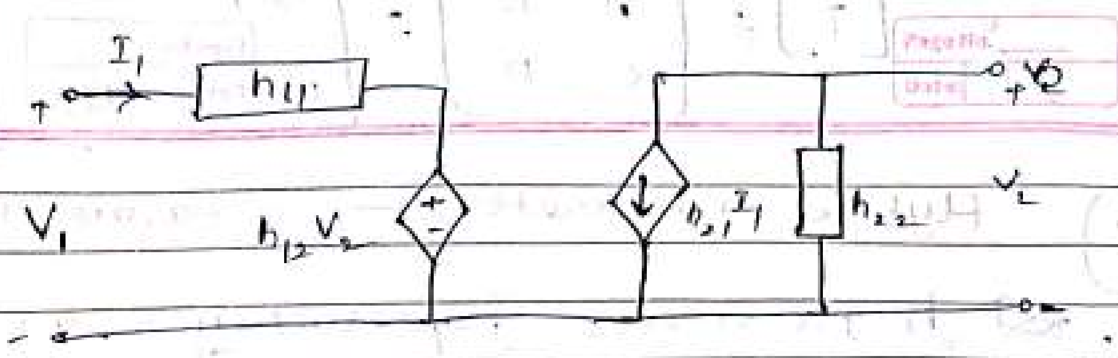
$$h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2 = 0}$$ → Input impedance

$$h_{21} = \left. \frac{I_2}{I_1} \right|_{V_2 = 0}$$ - forward Current gain

जब Input port को open circuit करते हैं ($I_1 = 0$)

$$h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1 = 0}$$ - reverse voltage gain

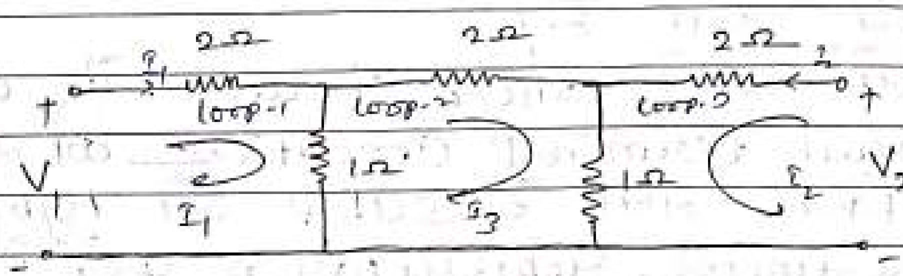
$$h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1 = 0}$$ - Output admittance.



h-parameter equivalent circuit

Nov-Dec 2016.

Q.



Calculate the h-parameter for given circuit.

$$V_1 = 2I_1 + 1(I_1 - I_3).$$

$$V_1 = 3I_1 - I_3 \quad \text{— (i)}$$

KVL in loop-2 :-

$$\cancel{2I_3} \; 4I_3 - I_1 + I_2 = 0 \quad \text{— (ii)}$$

KVL in loop-3 :-

$$V_2 = 3I_2 + I_3 \quad \text{— (iii)}$$

eqn (ii) को eqn (i) पर रखते है।

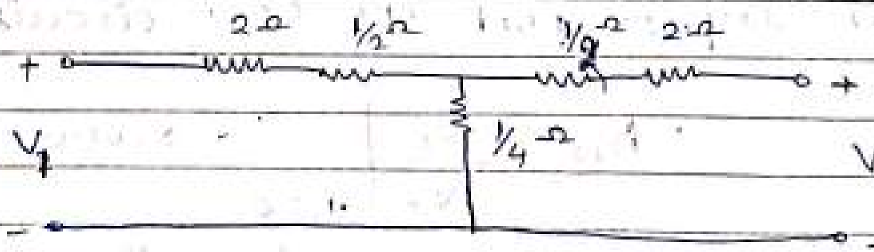
$$V_1 = 3I_1 - \left(\frac{I_1 - I_2}{4}\right)$$

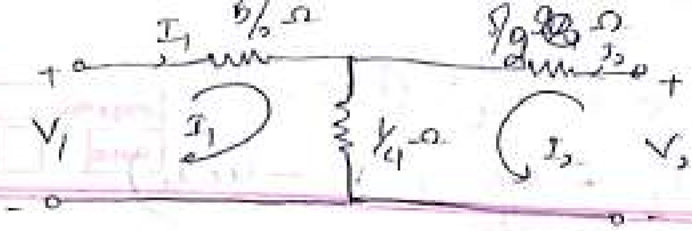
$3I_1 - \frac{I_1}{4}$

$V_1, V_2 = g(I_1, I_2)$

$$V_1 = \frac{11 I_1}{4} + \frac{I_2}{4}$$

Convert π to Y :-





KVL in loop 1, $V_1 = \frac{5}{2} I_1 + \frac{1}{4}(I_1 + I_2)$.

$V_1 = \frac{11}{4} I_1 + \frac{1}{4} I_2$ —— (i)

KVL in loop 2, $V_2 = \frac{10}{4} I_1 + \frac{6}{4} I_2$ —— (ii) $V_2 = \frac{5}{2} I_2 + \frac{1}{4}(I_1+I_2)$

Putting (ii) on eqn (i), $V_1 = \frac{11}{4} I_1 + \frac{1}{4}(4V_2 - 10 I_1)$ $V_1 = \frac{11}{4} I_1 + \frac{I_2}{4}$

$= \frac{11}{4} I_1 + V_2 - \frac{10}{4} I_1$

$$\boxed{V_1 = \frac{1}{4} I_1 + V_2}$$ —— (iii)

$4V_2 = 10 I_2 + I_1$

$$\boxed{I_2 = -\frac{1}{10} I_1 + \frac{4}{10} V_2}$$

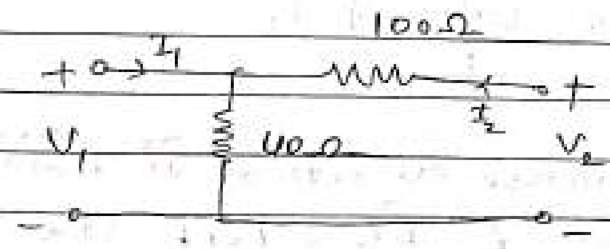
∴ Comparing with General eqn of h-param

$V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

$$\therefore h = \begin{bmatrix} 1/4 & 1 \\ -1/10 & 4/10 \end{bmatrix}$$

Q



$V_1 = 40 I_1 + 40 I_2$ —— (i)

$V_2 = 40 I_1 + 140 I_2$ —— (ii)

Putting eqn (ii) on eqn (i):-

$$V_1 = 40I_1 + \frac{40}{140}\left(V_2 - 40I_1\right)$$

$$\boxed{V_1 = \frac{200}{7}I_1 + \frac{2}{7}V_2}$$

$$I_2 = \frac{V_2 - 40I_1}{140}$$

$$\boxed{I_2 = -\frac{2}{7}I_1 + \frac{1}{40}V_2}$$

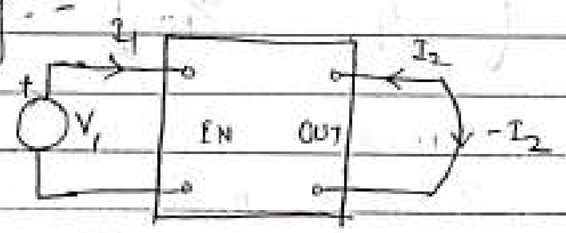
$$\therefore [h] = \begin{bmatrix} \frac{200}{7}\,\Omega & \frac{2}{7} \\ -\frac{2}{7} & \frac{1}{40}\,\mho \end{bmatrix}$$

Condition of Reciprocity & Symmetry in 2 Port:-

एक Network reciprocal होगा जब response और excitation के position को interchange कर दे फिर भी response और excitation का ratio बराबर होगा तो Network reciprocal कहलाता है।

∴ बिना port voltage या current को change किये यदि input और output port को change करते है तो वह Network symmetrical होता है।

Reciprocity:-



यहाँ I_2 को reverse direction में मानेंगे और output voltage V_2 को short कर देंगे।

Z-parameter Network equation,

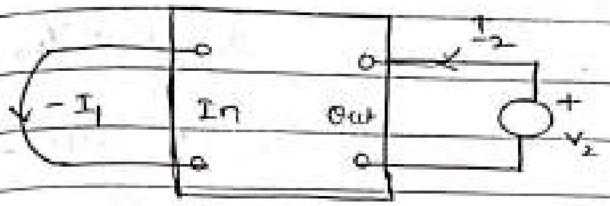
$$V_1 = Z_{11}I_1 - Z_{12}I_2 \quad \text{(i)}$$

$$0 = Z_{21}I_1 - Z_{22}I_2 \quad \text{(ii)}$$

. Equation ① और ⑪ से -

$$I_2 = \frac{V_1 Z_{21}}{Z_{11} Z_{22} - Z_{21} Z_{12}} \quad \text{—— (III)}$$

अब input और Output Voltage को change करेंगे -



$$0 = -Z_{11} I_1 + Z_{12} I_2 \quad \text{—— (IV)}$$

$$V_2 = -Z_{21} I_1 + Z_{22} I_2 \quad \text{—— (V)}$$

यह I_1 को opposite direction में मानेंगे और V_1 को short कर देंगे।

$$I_1 = \frac{V_2 Z_{12}}{Z_{11} Z_{22} - Z_{12} Z_{21}}$$

Assume $V_1 = V_2$,

$$\boxed{Z_{12} = Z_{21}}$$

Symmetry :- input port पर V apply करेंगे और o/p Port को open करेंगे

$$V = Z_{11} I_1 \quad \text{i.e} \quad Z_{11} = \frac{V}{I_1}\bigg|_{I_2 = 0}$$

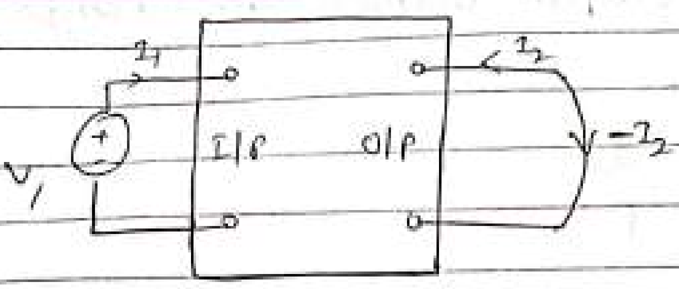
Again, Output port पर V apply करेंगे और input circuit को open करेंगे।

$$V = I_2 Z_{22} \Rightarrow Z_{22} = \frac{V}{I_2}\bigg|_{I_1 = 0} .$$

Symmetry Condition, $\frac{V}{I_1} = \frac{V}{I_2}$

$$\boxed{\therefore Z_{11} = Z_{22}}$$

Condition of Reciprocity in Y-parameter :-



$I_1 = y_{11}V_1 + y_{12}V_2$

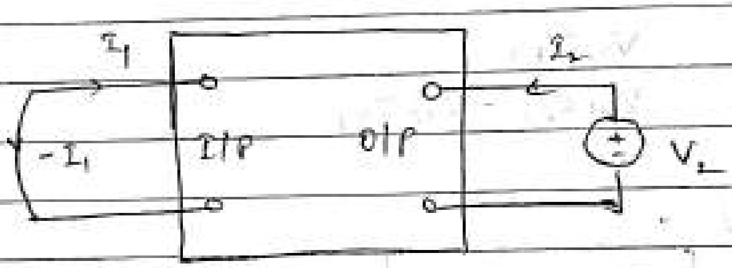
$I_2 = y_{21}V_1 + y_{22}V_2$

यदि $V_2 = 0$ और $-I_2$

$\therefore -I_2 = y_{21}V_1$

$\therefore y_{21} = \frac{-I_2}{V_1}\Big|_{V_2=0}$ — (i)

Now, input और output port को interchange करेंगे,



यदि $V_1 = 0$ और I_1 को opposite direction में flow होते हुए मानते हैं।

$\therefore -I_1 = y_{12}V_2$

$\therefore y_{12} = \frac{-I_1}{V_2}\Big|_{V_1=0}$ — (ii)

eqn (i) और (ii) से, $V_1 = V_2$

$$\therefore \boxed{y_{21} = y_{12}}$$

• Network का Y-parameter Symmetrical होगा यदि i/p और output port Interchange होगा बिना किसी current या Voltage के change के। यह तभी संभव है जब

$$\boxed{y_{11} = y_{22}}$$

Condition of Reciprocity in ABCD Parameter

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

यदि port 1 पर V_1 और output को short करते हैं, तो

$$V_1 = -BI_2$$

$$\frac{I_2}{V_1} = -\frac{1}{B} \quad \text{—— (1)}$$

यदि excitation और Response को interchange करते हैं, port -2 पर V_2 और short circuit current I_1 port -1 पर मिलेगा।

$$0 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

$$I_2 = \frac{A}{B}V_2 \; , \; I_1 = CV_2 - \frac{AD}{B}V_2$$

$$I_2 = V_2\left(\frac{BC - AD}{B}\right)$$

or, $$\frac{I_2}{V_2} = \frac{BC - AD}{B} \quad \text{—— (1)}$$

जब $V_1 = V_2$ होगा,

$$-\frac{1}{\cancel{B}} = \frac{BC - AD}{\cancel{B}}$$

$$\boxed{\therefore \; \boxed{AD - BC} = 1}$$ — Condition for reciprocity

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = 1$$

For Symmetry,

$$Z_1 = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \left.\frac{AV_2 - BI_2}{CV_2 - DI_2}\right|_{I_2=0} = \frac{A}{C}$$

$$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0} = \frac{D}{C}$$

∴ For symmetry in Z - N/ω, $Z_{11} = Z_{22}$

$$\therefore \frac{A}{C} = \frac{D}{C}$$

$$\boxed{A = D}$$

Condition for ~~Symmetry~~ Reciprocity in h - Parameter:

$$\boxed{h_{12} = -h_{21}}$$

Symmetry, $\boxed{h_{11} \cdot h_{22} - h_{21} \cdot h_{12} = 1}$

Parameter	Condn for Reciprocity	Condition for Symmetry
Z	$Z_{12} = Z_{21}$	$Z_{11} = Z_{22}$
y	$y_{12} = y_{21}$	$y_{11} = y_{22}$
ABCD	$AD - BC = 1$	$A = D$
h	$h_{12} = -h_{21}$	$h_{11} \cdot h_{22} - h_{21} \cdot h_{12} = 1$

Inter-relation Between Different Parameter

	[Z]	[Y]	[h]	[T]
[Z]	$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{y_{22}}{\Delta y} & \frac{-y_{12}}{\Delta y} \\ \frac{-y_{21}}{\Delta y} & \frac{y_{11}}{\Delta y} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ -\frac{h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{A}{C} & \frac{\Delta T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$
[Y]	$\begin{bmatrix} \frac{Z_{22}}{\Delta Z} & -\frac{Z_{12}}{\Delta Z} \\ -\frac{Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix}$	$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{h_{11}} & -\frac{h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{D}{B} & -\frac{\Delta T}{B} \\ -\frac{1}{B} & \frac{A}{B} \end{bmatrix}$
[h]	$\begin{bmatrix} \frac{\Delta Z}{Z_{22}} & \frac{Z_{21}}{Z_{22}} \\ -\frac{Z_{21}}{Z_{22}} & \frac{1}{Z_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{y_{11}} & -\frac{y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta y}{y_{11}} \end{bmatrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{B}{D} & \frac{\Delta T}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{bmatrix}$
[T]	$\begin{bmatrix} \frac{Z_{11}}{Z_{21}} & \frac{\Delta Z}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{bmatrix}$	$\begin{bmatrix} -\frac{y_{22}}{y_{21}} & -\frac{1}{y_{21}} \\ \frac{-\Delta y}{y_{21}} & -\frac{y_{11}}{y_{21}} \end{bmatrix}$	$\begin{bmatrix} -\frac{\Delta h}{h_{21}} & -\frac{h_{11}}{h_{21}} \\ -\frac{h_{22}}{h_{21}} & -\frac{1}{h_{21}} \end{bmatrix}$	$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$

$\Delta Z = Z_{11} Z_{22} - Z_{21} Z_{12}$

Q. The Z-parameter of a two port Network are $Z_{11} = 20\,\Omega$, $Z_{22} = 30\,\Omega$, $Z_{12} = Z_{21} = 10\,\Omega$ find y & ABCD Parameter, h-parameter.

$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

$$= \begin{bmatrix} 20 & 10 \\ 10 & 30 \end{bmatrix}$$

$\therefore \Delta Z = 600 - 100 = 500.$

(1) Find y-Parameter:-

$$y_{11} = \frac{z_{22}}{\Delta z} = \frac{30}{500} = \frac{3}{50}\ \mho$$

$$y_{12} = \frac{-z_{12}}{\Delta z} = \frac{-10}{500} = \frac{-1}{50}\ \mho$$

$$y_{21} = \frac{-z_{21}}{\Delta z} = \frac{-10}{500} = -\frac{1}{50}\ \mho$$

$$y_{22} = \frac{z_{11}}{\Delta z} = \frac{20}{500} = \frac{2}{50} = \frac{1}{25}\ \mho$$

$$y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} \frac{3}{50} & -\frac{1}{50} \\ -\frac{1}{50} & \frac{1}{25} \end{bmatrix}\ \mho$$

(ii) To find ABCD Parameters:—

$$A = \frac{z_{11}}{z_{21}} = \frac{20}{10} = 2$$

$$B = \frac{\Delta z}{z_{21}} = \frac{500}{100} = 50\ \Omega$$

$$C = \frac{1}{z_{21}} = \frac{1}{10} = 0.1\ \mho$$

$$D = \frac{z_{22}}{z_{21}} = \frac{30}{10} = 3$$

$$[T] = \begin{bmatrix} 2 & 50 \\ 0.1 & 3 \end{bmatrix}$$

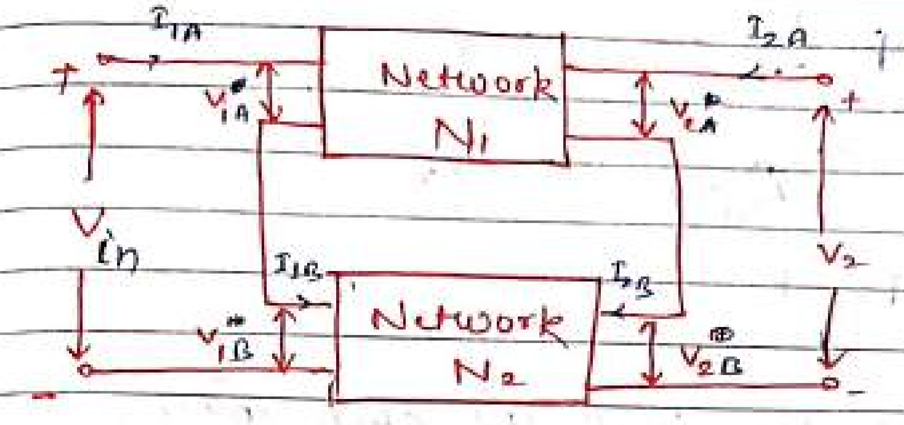
(iii) $$h_{11} = \frac{\Delta z}{z_{22}} = \frac{500}{30} = \frac{50}{3}\ \Omega$$

$$h_{12} = \frac{z_{21}}{z_{22}} = \frac{10}{30} = \frac{1}{3}$$

$$h_{21} = \frac{-z_{21}}{z_{22}} = \frac{-10}{30} = -\frac{1}{3}$$

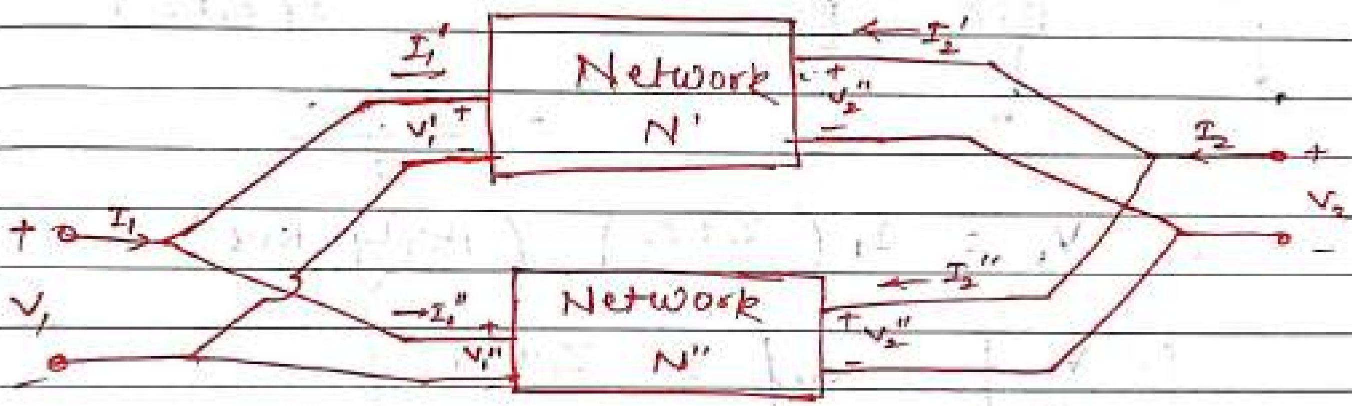
$$h_{22} = \frac{1}{z_{22}} = \frac{1}{30}\ \mho$$

Series Connection of two port:-



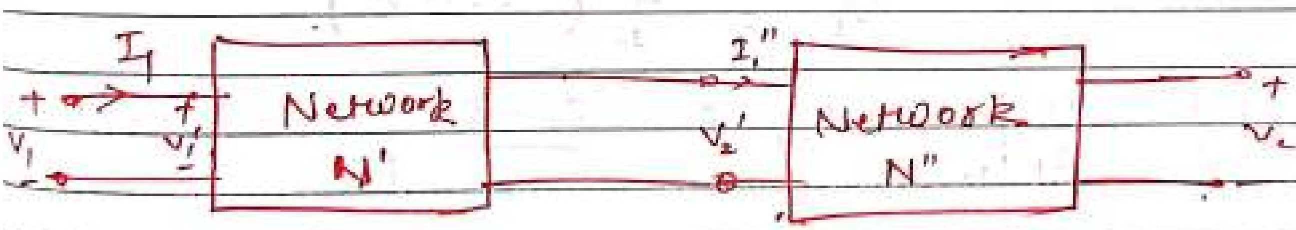
$$[Z] = [Z_1] + [Z_2] \text{ (Proof)}$$

parallel Connection of two port:-



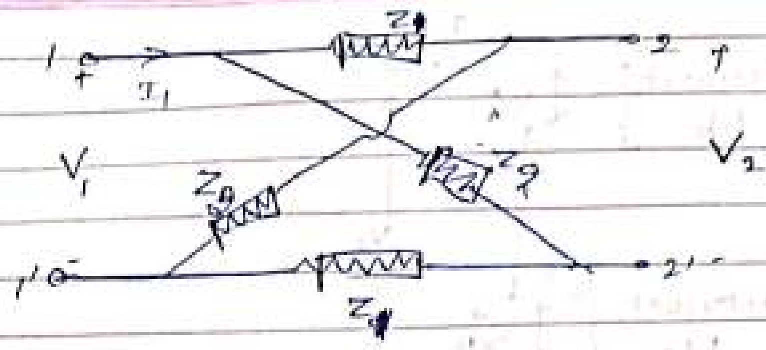
$$[y] = [y_1] + [y_2]$$

Cascade Connection of two port:-

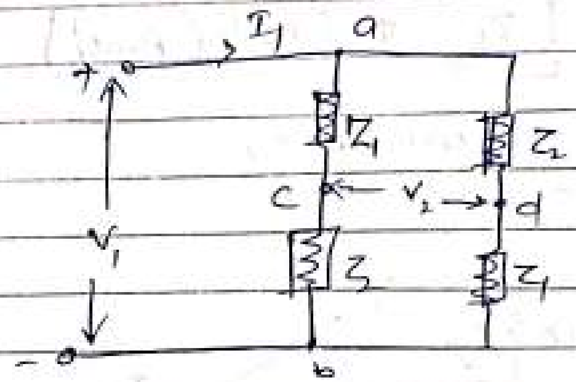


$$[A] = [ABCD_1] + [ABCD_2]$$

Lattice Network: —



Step-I :- Output terminal को open करेंगे.



Total impedance =

$$= \frac{(Z_1+Z_2)(Z_1+Z_2)}{2(Z_1+Z_2)}$$

$$= \frac{Z_1+Z_2}{2}.$$

$$\therefore V_1 = I_1\left(\frac{Z_1+Z_2}{2}\right). \text{ (Apply KVL)}.$$

$$\boxed{\frac{V_1}{I_1}\bigg|_{I_2=0} = Z_{11} = \frac{Z_1+Z_2}{2}}$$

$$V_2 = V_c - V_d = \frac{I_1 \cdot \cancel{(Z_1+Z_2)}}{2\cancel{(Z_1+Z_2)}} * Z_2 - Z_1 \frac{I_1}{2}$$

$$V_2 = \frac{I_1}{2}(Z_2 - Z_1).$$

$$\therefore \boxed{\frac{V_2}{I_1}\bigg|_{I_2=0} = Z_{21} = \frac{Z_2-Z_1}{2}}$$

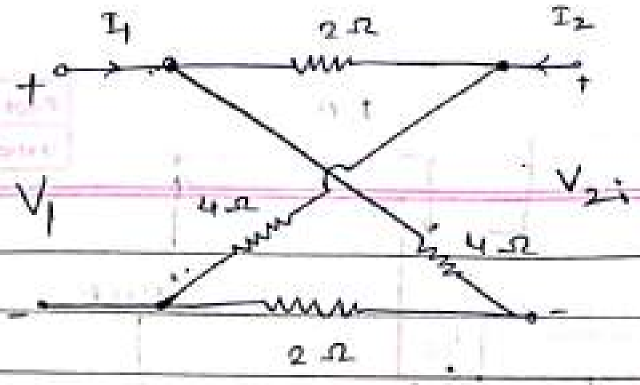
~~Step-II:- Now input terminal को open करेंगे~~ /

Given N/W is symmetrical so $Z_{12} = Z_{21}$

और $Z_{22} = Z_{11}$

$$\boxed{\begin{aligned} \therefore Z_{11} = Z_{22} &= \frac{Z_1+Z_2}{2} \\ Z_{21} = Z_{12} &= \frac{Z_2-Z_1}{2} \end{aligned}}$$

Q.



find ① Z-paramet, Y-Paramete, h-Paramete, ABCD parameter,

We know that, For lattice Network,

$$Z_{11} = Z_{22} = \frac{Z_1 + Z_2}{2} = \frac{2+4}{2} = 3\Omega$$

$$Z_{21} = Z_{12} = \frac{Z_2 - Z_1}{2} = \frac{2}{2} = 1\Omega$$

$$Z = \begin{bmatrix} 3\Omega & 1\Omega \\ 1\Omega & 3\Omega \end{bmatrix}$$

$$y = [Z]^{-1} = \frac{1}{8}\begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 3/8\,℧ & -1/8\,℧ \\ -1/8\,℧ & 3/8\,℧ \end{bmatrix}$$

h-parameter,

$$h_{11} = \frac{\Delta Z}{Z_{22}} = 8/3\ \Omega$$

$$h_{22} = \frac{1}{Z_{22}} = 1/3\ ℧$$

$$h_{12} = \frac{Z_{12}}{Z_{22}} = 1/3$$

$$h_{21} = \frac{-Z_{21}}{Z_{22}} = -1/3$$

ABCD parameter :—

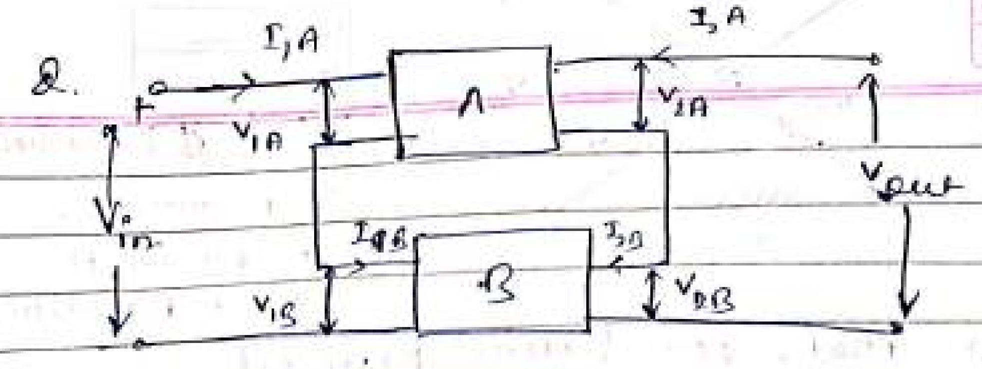
$$A = \frac{Z_{11}}{Z_{21}} = \frac{3}{1} = 3$$

$$B = \frac{\Delta Z}{Z_{21}} = \frac{8}{1} = 8$$

$$C = 1/Z_{21} = 1$$

$$D = \frac{Z_{22}}{Z_{21}} = 3/1 = 3.$$

2.



For N/W A,

$$V_{1A} = Z_{11A} I_{1A} + Z_{12A} I_{2A}$$

$$V_{2A} = Z_{21A} I_{1A} + Z_{22A} \cdot I_{2A}$$

For Network B,

$$V_{1B} = Z_{11B} I_{1B} + Z_{12B} I_{2B}$$

$$V_{2B} = Z_{21B} I_{1B} + Z_{22B} I_{2B}$$

$$I_1 = I_{1A} = I_{1B}$$

$$I_2 = I_{2A} = I_{2B}.$$

$$V_2 = V_{2A} + V_{2B}.$$

However, $V_1 = V_{1A} + V_{1B}.$

$$= (Z_{11A} I_{1A} + Z_{12A} I_{2A}) + (Z_{11B} I_{1B} + Z_{12B} I_{2B}$$

$$= I_1 (Z_{11A} + Z_{11B}) + I_2 (Z_{12A} + Z_{12B})$$

& $V_2 = V_{2A} + V_{2B}$

$$= (Z_{21A} I_{1A} + Z_{22A} I_{2A}) + (Z_{21B} I_{1B} + Z_{22B} I_{2B})$$

$$V_2 = I_1 (Z_{21A} + Z_{21B}) + I_2 (Z_{22B} + Z_{22A})$$

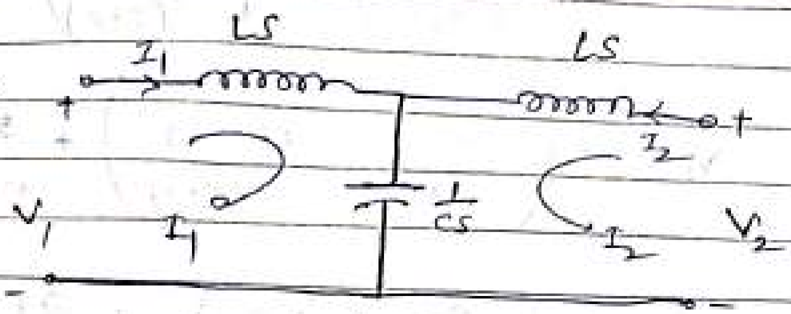
Thus we get series connected two numbers two port network

$$V_1 = (Z_{11A} + Z_{11B}) I_1 + (Z_{12A} + Z_{12B}) I_2$$

$$V_2 = (Z_{21A} + Z_{21B}) I_1 + (Z_{22A} + Z_{22B}) I_2$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11A} + Z_{11B} & Z_{12A} + Z_{12B} \\ Z_{21A} + Z_{21B} & Z_{22A} + Z_{22B} \end{bmatrix}$$

Q.



find Z-parameter

$$V_1 = LS I_1 + \frac{1}{CS}(I_1 + I_2)$$

$$V_1 = I_1\left(LS + \frac{1}{CS}\right) + \frac{I_2}{CS}$$

$$V_1 = I_1\left(\frac{S^2LC+1}{CS}\right) + \frac{I_2}{CS} \quad \text{--- } (1)$$

$$V_2 = LS I_2 + \frac{1}{CS}(I_2 + I_1)$$

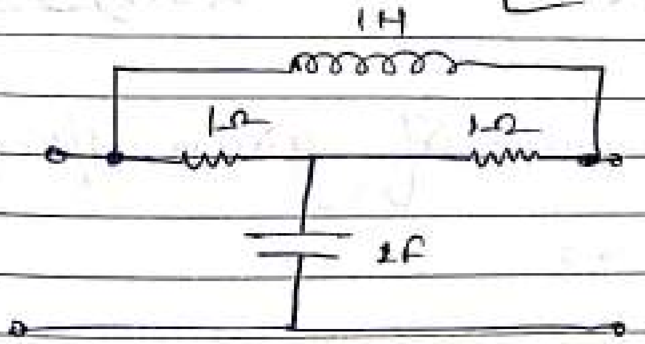
$$= \frac{I_1}{SC} + I_2\left(SL + \frac{1}{SC}\right)$$

$$= \frac{1}{CS} I_1 + I_2\left(\frac{S^2LC+1}{SC}\right) \quad \text{--- } (11)$$

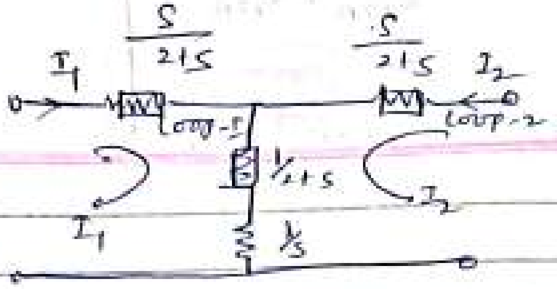
Comparing eq^n (1) & (11) -

$$[Z] = \begin{bmatrix} \frac{S^2LC+1}{CS} & \frac{1}{CS} \\ \frac{1}{CS} & \frac{S^2LC+1}{SC} \end{bmatrix}$$

Q.



find z-parameter.



$$V_1 = \left(\frac{S}{2+S}\right) I_1 + \left(\frac{2s+2}{S(s+2)}\right)(I_1+I_2)$$

$$V_1 = I_1\left(\frac{S}{S+2} + \frac{2(S+1)}{S(S+2)}\right) + \frac{2(S+1)}{S(S+2)} I_2$$

$$V_1 = I_1\left(\frac{S^2+2S+2}{S(S+2)}\right) + \frac{2(S+1)}{S(S+2)} I_2 \quad \text{— (i)}$$

KVL in loop-2 :-

$$\frac{S}{2+S} I_2 + \frac{2(S+1)}{S(S+2)} \cdot (I_2 + I_1) = V_2$$

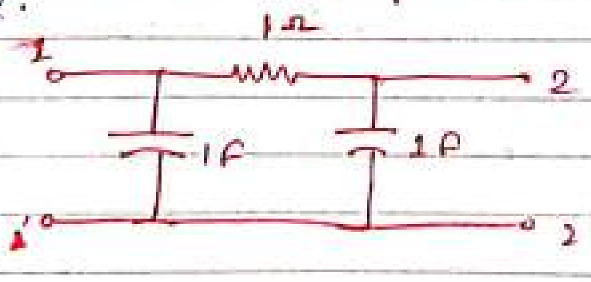
$$V_2 = \frac{2(S+1)}{S(S+2)} I_1 + I_2 \times \left[\frac{S}{S+2} + \frac{2(S+1)}{S(S+2)}\right]$$

$$= \frac{2(S+1)}{S(S+2)} I_1 + \left(\frac{S^2+2S+2}{S(S+2)}\right) I_2 . \quad \text{— (ii)}$$

Comparing eq^n (i) & (ii),

$$[Z] = \begin{bmatrix} \frac{S^2+2S+2}{S(S+2)} & \frac{2(S+1)}{S(S+2)} \\ \frac{2(S+1)}{S(S+2)} & \frac{S^2+2S+2}{S(S+2)} \end{bmatrix}$$

Q. Find ABCD Parameter of given N/w ?



Chapter-V Laplace Transform

- Laplace transform linear differential equation को solve करने का सबसे powerful tool है।
- Initial condition को भी laplace transform consider करता है, इसलिए इसे बहुत ही आसानी से किसी भी linear differential equation को solve करने के लिए use किया जाता है।
- Non-Homogeneous differential equation का solution भी laplace transform की मदद से आसानी से निकाला जाता है।

Laplace transform :-

$$\boxed{F(s) = L[f(t)] = \int_0^{\infty} f(t)\, e^{-st}\, dt.}$$

जहाँ F(S) एक complex frequency domain या S-domain में है।

और $S = \sigma + j\omega$.

σ = attenuation

ω = angular frequency

Inverse Laplace transform :- इसके द्वारा हम S-domain से time domain में ला सकते हैं।

$$\boxed{L^{-1}[F(s)] = f(t) = \frac{1}{2\pi j} \int_{\sigma_0 - j\omega}^{\sigma_0 + j\omega} F(s)\, e^{st}\, ds}$$

(i) Laplace transform of derivative:-

$$L \frac{df(t)}{dt} = sF(s) - f(0^+).$$

for n^{th} derivative.

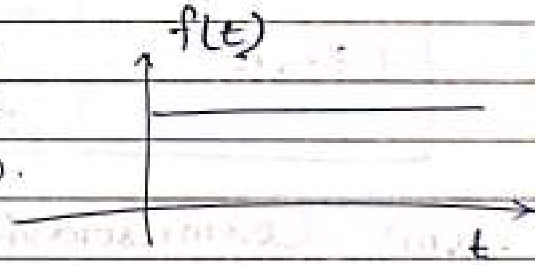
$$L[f^n(t)] = s^n F(s) - s^{n-1} f(0^+) - s^{n-2} f'(0^+) - \cdots - f^{n-1}(0)$$

(ii) Laplace transform of Integral $\int f(t)dt$:-

$$L\left[\int f(t)dt\right] = \frac{1}{s} F(s) + \frac{1}{s} \int F(t)dt \Big|_{t=0^+}$$

(iii) Unit step function:—

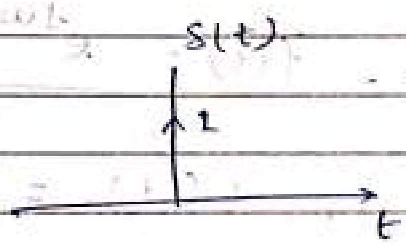
$$u(t) = \begin{cases} 0, & t \leq 0 \\ 1, & t > 0. \end{cases}$$



$$\boxed{\therefore L[u(t)] = \frac{1}{s}.}$$

(iv) Unit impulse function:—

$$\delta(t) = \lim_{\Delta t \to 0} \frac{u(t) - u(t-\Delta t)}{\Delta t}.$$

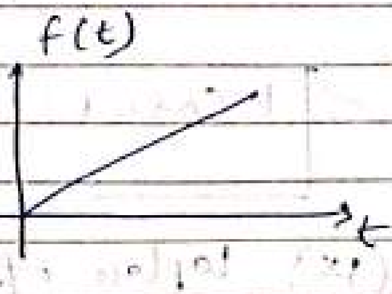


$$\cancel{\delta(t) =} \boxed{L\,\delta(t) = 1}$$

(v) Unit ramp function:—

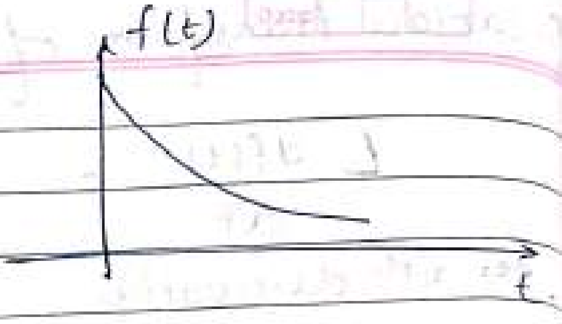
$$f(t) = t, \quad 0 < t < \infty.$$

$$0, \quad -\infty < t < 0$$



$$\boxed{\therefore L f(t) = \frac{1}{s^2}}$$

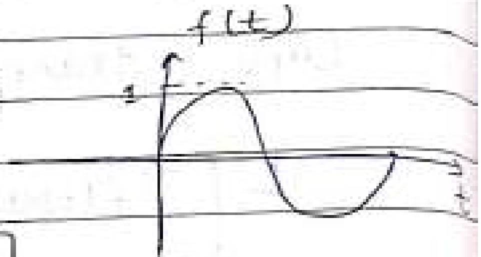
(VI) Exponential function,



$$f(t) = e^{-\alpha t}$$

$$\therefore Lf(t) = \frac{1}{s+\alpha}.$$

(VII) Sinusoidal function,

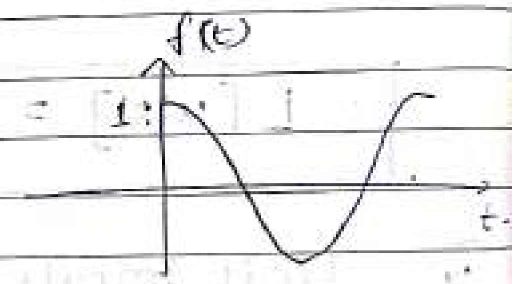


$$f(t) = \sin\omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

$$Lf(t) = \frac{1}{2j}\left[\frac{1}{s-j\omega} - \frac{1}{s+j\omega}\right]$$

$$\boxed{L\sin\omega t = \frac{\omega}{s^2+\omega^2}}$$

(VIII) Cosinusoidal function



$$f(t) = \cos\omega t$$

$$f(t) = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

$$\therefore Lf(t) = \frac{1}{2}\left[\frac{1}{s-j\omega} + \frac{1}{s+j\omega}\right]$$

$$= \frac{1}{2}\left[\frac{s+\cancel{j\omega} + s-\cancel{j\omega}}{s^2+\omega^2}\right]$$

$$\boxed{L\cos\omega t = \frac{s}{s^2+\omega^2}}$$

(IX) Laplace of t^n:-

$$\boxed{Lt^n = \frac{n!}{s^{n+1}}}$$

(x) Laplace transform of $e^{-\alpha t} \cdot \sin\omega t$:-

$$:f(t) = e^{-\alpha t} \cdot \left[\frac{e^{j\omega t} - e^{-j\omega t}}{2j} \right]$$

$$= \frac{1}{2j} \left[e^{-(j\omega+\alpha)t} - e^{-(j\omega+\alpha)t} \right]$$

$$= \frac{1}{2j} \left[\frac{1}{s+\alpha-j\omega} - \frac{1}{s+\alpha+j\omega} \right]$$

$$\boxed{Lf(t) = \frac{\omega}{(s+\alpha)^2 + \omega^2}}$$

(xi) $\boxed{Lf(t-T) = e^{-sT} F(s)}$ → Displacement theorem

(Q) $f(t) = 1 - e^{-at}$, find Laplace transform.

$$Lf(t) = \int_0^\infty (1 - e^{-at}) e^{-st} dt$$

$$= \int_0^\infty e^{-st} dt - \int_0^\infty e^{-(a+s)t} dt$$

$$= \left[\frac{e^{-st}}{-s} \right]_0^\infty - \left[\frac{e^{-(a+s)t}}{-(a+s)} \right]_0^\infty$$

$$= -\frac{1}{s}(0-1) + \frac{1}{a+s}(0-1]$$

$$= \frac{1}{s} - \frac{1}{a+s}$$

$$= \frac{s+a-s}{s(s+a)} = \frac{a}{s(s+a)}$$

Q. find inverse laplace transform of

$\frac{3+2S}{S(S+1)}$ $F(S) = \frac{10^4}{S(S+250)}$

∴ from partial fraction,

$$\frac{10^4}{S(S+250)} = \frac{A}{S} + \frac{B}{(S+250)}$$

$$\frac{10^4}{S(S+250)} = \frac{AS + 250A + BS}{S(S+250)}$$

$\therefore A + B = 0, \quad 250A = 10^4$

$A = \frac{10000}{250} = 40$

$\boxed{B = -40}$

$$\therefore F(S) = \frac{40}{S} - \frac{40}{S+250}$$

$$L^{-1}[F(S)] = L^{-1}\left[\frac{40}{S} - \frac{40}{S+250}\right]$$

$$\boxed{f(t) = 40 - 40 \times e^{-250t}}$$

April-may 2016

Q find Inverse laplace transform of

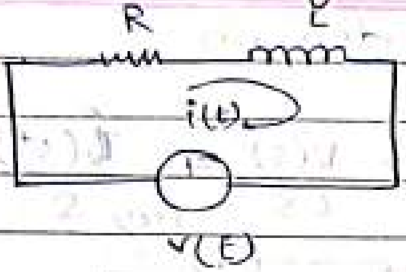
$$\frac{3+2S}{S(S+1)}.$$

Q. $F(S) = \frac{250}{(S+100)(S+50)}$ $\left(\text{Ans} = -5e^{-100t} + 5e^{-50t}\right)$

Q. $F(S) = \frac{250}{(S^2+625)(S+2)}$ $\left(0.4e^{-2t} - \frac{5}{(25+2j)}e^{-j25t} - \frac{5}{25-2j}e^{j25t}\right)$

Q. $F(S) = \frac{S^2+3S+1}{S(S^2+3S+2)}$ $\left(f(t) = \frac{1}{2} - \frac{1}{2}e^{-2t} + e^{-t}\right)$

Application of laplace transform in Electric circuit :-



$$Ri(t) + L\frac{di(t)}{dt} = v(t) \quad \text{--- (1)}$$

time domain को laplace domain में convert करते हैं तो

$$i(t) \longrightarrow I(s).$$

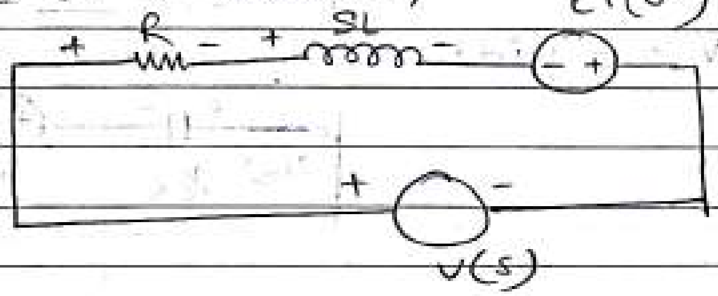
$$\frac{di(t)}{dt} \longrightarrow \left[sI(s) - i(0^+)\right]$$

$\therefore$ Equation (1) का Laplace दोनो तरफ लेने पर

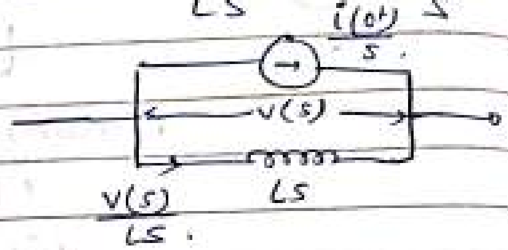
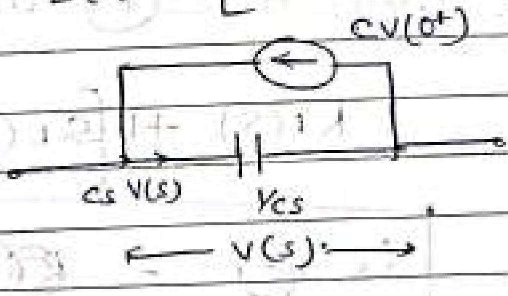
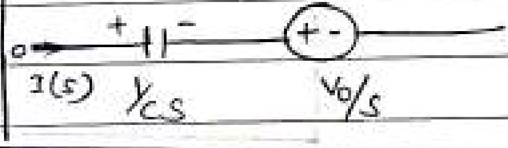
$$RI(s) + L\left[sI(s) - i(0^+)\right] = V(s)$$

$$\boxed{\therefore V(s) = RI(s) + LsI(s) - Li(0^+)}$$

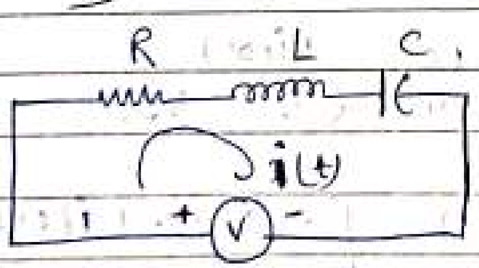
R-L circuit in s-domain,



Time domain		S-domain
(1) $i(t)$, R, + $v(t)$ −	$v(t) = Ri(t)$	$V(s) = RI(s)$
(2) i, L, + $v(t)$ −	$v(t) = L\frac{di}{dt}$	Ls, $Li(0^+)$, $I(s)$, $V(s) =$ $V(s) = \left[sLI(s) - Li(0^+)\right]$

time domain	s-domain
(iii) Current in inductor, $i(t) = \frac{1}{L}\int_0^t V(t)\,dt + i(0^+)$	$I(s) = \frac{V(s)}{LS} + \frac{i(0^+)}{S}$ 
(iv) current in Capacitor. $i(t) = C\frac{dv(t)}{dt}$	$I(s) = C\left[sV(s) - V(0^+)\right]$ 
(v) voltage in Capacitor. $V(t) = V_0 + \frac{1}{C}\int i\,dt$	$V(s) = \frac{V_0}{S} + \frac{1}{C}\frac{I(s)}{S}$ 

(Q) A time dependent voltage V(t) is applied to a Series Connection of RLC Network. find s-domain impedance & current. Draw s-domain circuit.



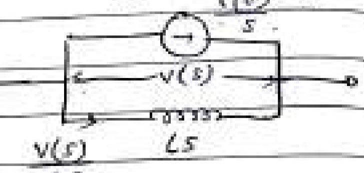
$$v(t) = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t$$

Corresponding laplace transfor

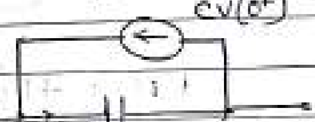
$$V(s) = R\,I(s) + SLI(s) - LI(0^+) + \frac{1}{SC}I(s) + \frac{V_0}{S}$$

s-domain

$I(s) = \frac{V(s)}{Ls} + \frac{i(0^+)}{s}$



$I(s) = C[sV(s) - V(0^+)]$



$V(s) = \frac{V_0}{s} + \frac{1}{C}\frac{I(s)}{s}$

V(t) is applied.

...f RLC Network.

& current

$Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt$

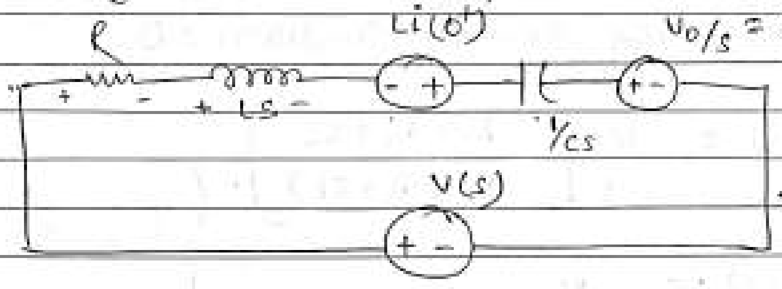
...ing laplace transform

$\ldots + \frac{1}{SC} I(s) + \frac{V_0}{S}$

$I(s)\left[R + SL + \frac{1}{CS}\right] = V(s) - \frac{Q_0}{CS} + LI(0^+)$ $\left(\because V_0 = \frac{Q_0}{C}\right)$

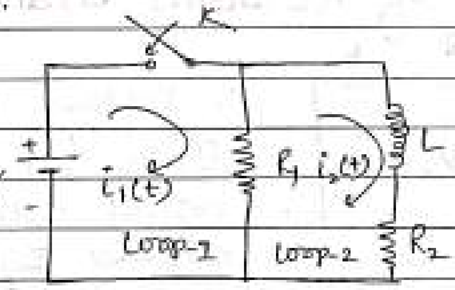
$$\boxed{\therefore\ I(s) = \frac{V(s) - Q_0/CS + LI(0^+)}{\left(R + LS + \frac{1}{CS}\right)}}$$

$\left\{Z = R + LS + \frac{1}{CS}\right\}$



→ S-domain circuit

Q) A two mesh network is shown obtain the expression for $I_1(s)$ & $I_2(s)$ when the switch is closed.



Apply KVL in loop-1 & loop-2 :-

$R_1[i_1(t) - i_2(t)] = V$. ——— (i)

& $R_2 i_2(t) + R_1 i_2(t) - R_1 i_1(t) + L\frac{di_2(t)}{dt} = 0$. (ii)

Taking laplace transform of eqn (i) & (ii) :-

$R_1[I_1(s) - I_2(s)] = \frac{V}{S}$. ——— (iii)

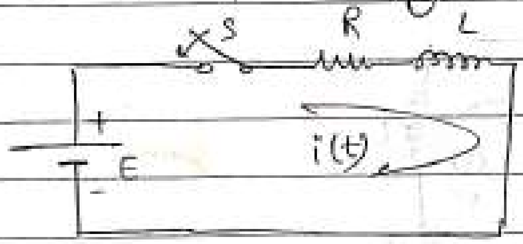
$R_2 I_2(s) + R_1 I_2(s) - R_1 I_1(s) + LS\, I_2(s) - LI_2(0^+) = 0$ (iv) from eq^n (III) & (IV), we can write,

$$\begin{bmatrix} R_1 & -R_1 \\ -R_1 & R_1+R_2+SL \end{bmatrix} \begin{bmatrix} I_1(S) \\ I_2(S) \end{bmatrix} = \begin{bmatrix} V/S \\ 0 \end{bmatrix}$$

∴ Assuming Initial Condition = 0.

$$\left.\begin{aligned} \therefore I_1(S) &= \frac{V}{S}\left[\frac{R_1+R_2+SL}{R_1(R_2+SL)}\right] \\ I_2(S) &= \frac{V}{S}\cdot\frac{1}{(R_2+SL)} \end{aligned}\right\}$$

- Step response of RL series Circuit :—



∴ Voltage $Eu(t)$ को Series R-L network पर लगाया जाता | Circuit को initially Zero Condition में चालू किया जाता

$$Eu(t) = Ri(t) + L\frac{di}{dt}.$$

Taking Laplace transform on both side,

$$\frac{E}{S} = RI(S) + LSI(S) - LI(0^+).$$

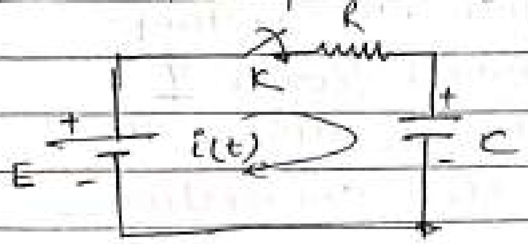
$$\text{or,} \quad I(S) = \frac{E/L}{LS\left(S+\frac{R}{L}\right)}$$

$$I(S) = \frac{E/R}{SL} + \frac{-E/R}{S+R/L}$$

$$I(S) = \frac{E}{R}\cdot\frac{1}{S} - \frac{E}{R}\left(\frac{1}{S+R/L}\right)$$

$$\boxed{i(t) = \frac{E}{R} - \frac{E}{R}e^{-R/L\,t}}$$ Check

- Step response of RC series circuit :-



माना कि capacitance है 'C' के पास initially Q_0 charge है। Now switch K को बंद किया गया, तब circuit में बहने वाले current का equation

$$Eu(t) = Ri(t) + \frac{1}{C}\int i(t)\,dt$$

Taking laplace on both side,

$$\frac{E}{s} = RI(s) + \left[\frac{1}{Cs}I(s) + \frac{Q_0}{Cs}\right]$$

$$\frac{E}{s} - \frac{Q_0}{Cs} = RI(s) + \frac{I(s)}{Cs}$$

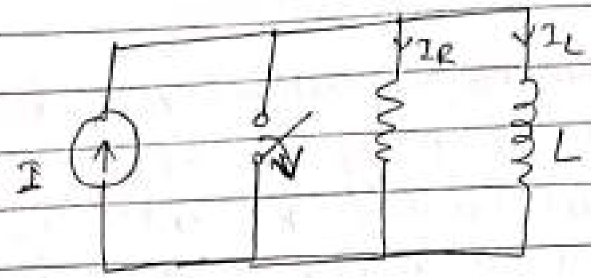
$$I(s) = \frac{\frac{E}{s} - \frac{Q_0}{Cs}}{\left(R + 1/Cs\right)}$$

$$= \frac{1/s\left(E - \frac{Q_0}{C}\right)}{\left(R + 1/Cs\right)} = \frac{E - Q_0/C}{s\left(R + 1/Cs\right)}$$

Or, $I(s) = \left(\frac{E}{R} - \frac{Q_0}{RC}\right)\left(\frac{1}{s + \frac{1}{RC}}\right)$

$$\boxed{I(t) = \left(\frac{E}{R} - \frac{Q_0}{RC}\right)e^{-1/RC\,t}}$$

Q. Step current Response of a RL Parallel circuit



माना कि Constant current Source I को R और L के Parallel connection पर apply किया जाता है।

$$RI_R = L\frac{dI_L}{dt} \quad \text{---(i)}$$

& $$I = I_R + I_L \quad \text{---(ii)}$$

multiplying equation (i) with R,

$$IR = RI_R + RI_L$$

$$IR = L\frac{dI_L}{dt} + RI_L \quad (\text{from eq}^n$$

for step input,

$$\frac{I_0(s)}{s} \cdot R = LSI_L(s) + RI_L(s) \quad \left(\begin{matrix}\text{Initial} \\ \text{Condition}\end{matrix}\right.$$

$$\frac{I(s)}{s} \cdot R = I_L(s)\left[LS + R\right]$$

$$I(s) = \frac{s(R+LS)}{R} I_L(s)$$

$$\therefore I_L(s) = I(s) \cdot \frac{R}{s(R+LS)}$$

$$I_L(s) = I(s)\left[\frac{1}{S} - \frac{1}{R+LS}\right]$$

$$\therefore I_L(s) = I(s)\left[\frac{1}{s} - \frac{1}{s+R/L}\right]$$

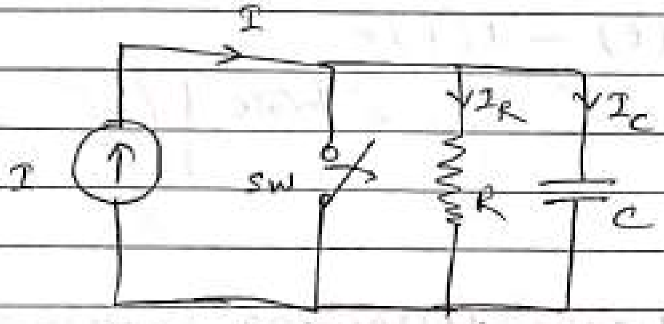
$$\boxed{i_L(t) = i(t)\left[1 - e^{-(R/L)t}\right]}$$

$$\therefore i_R(t) = i - i_L(t)$$

$$= \cancel{i} - \cancel{i} + i e^{-R/L t}$$

$$\boxed{i_R(t) = i \cdot e^{-R/L t}}$$

- Step current Response of RC parallel Circuit –



जब switch को open करते है तो current R & C circuit मे flow करने लगता है।

$$I = i_R + i_C \quad \text{—— (I)}$$

$$\& \quad R i_R = \frac{1}{C}\int_0^t i_C\,dt \quad (\because \text{Voltage same}) \quad \text{—— (II)}$$

Multiplying eq^n (1) with R,

$$RI = R i_R + R i_C$$

$$= \frac{1}{C}\int_0^t i_C\,dt + R i_C$$

Taking laplace transform,

$$R\frac{I(s)}{s} = \frac{1}{CS} I_C(s) + R I_C(s).$$

$$\frac{R}{s} I(s) = I_C(s)\left[\frac{1}{CS} + R\right].$$

$$\frac{R}{s} I(s) = I_C(s) = \frac{R}{s} I(s) \times \frac{CS}{1 + RCS}.$$

$$I_c(s) = I(s) \cdot \frac{1}{\left[s + \frac{1}{RC}\right]}.$$

Taking laplace inverse,

$$\boxed{i_c(t) = i(t) \cdot \left[e^{-t/RC}\right].}$$

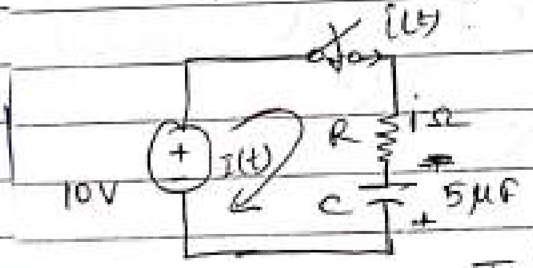
above equation is Capacitor current at $t = 0^+$.

$$\therefore\ i_R(t) = i(t) - i_c(t)$$

$$= i(t) - i(t)e^{-t/RC}$$

$$i_R(t) = i(t)\left[1 - e^{-t/RC}\right]$$

Q. find $i(t)$ in given figure following switching at $t=0$. Assume initial charge on Capacitor $250\mu C$.



Apply KVL in given circuit,

$$10 = Ri(t) + \frac{1}{C}\int i(t)dt$$

Taking laplace transform,

$$10 = RI(s) + \left[\frac{1}{CS} I(s) + \frac{Q(0)}{CS}\right]$$

Q = CV

$$Q_0 = -250 \times 10^{-6}\, C$$

$$\frac{10}{s} = I(s) + \frac{1}{5\times10^{-6}s} I(s) + \frac{250\times10^{-6}}{5\times10^{-6}s}$$

$$I(s) + \frac{I(s)}{5\times10^{-6}s} = \frac{10}{s} + \frac{50}{s} = \frac{60}{s}$$

$$I(s)\left[1 + \frac{2\times10^{5}}{s}\right] = \frac{60}{s}.$$

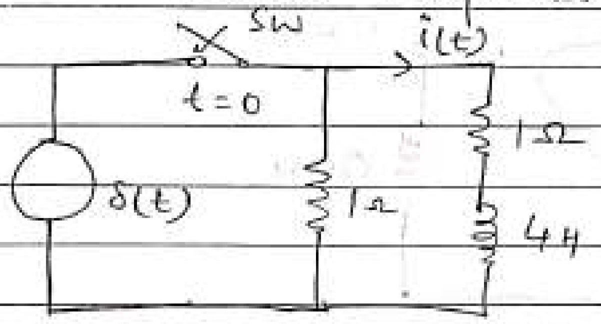
$$I(s) = \frac{60 \cdot s}{s(s + 2\times10^{5})}$$

$$I(s) = \frac{60}{s+2\times10^5}.$$

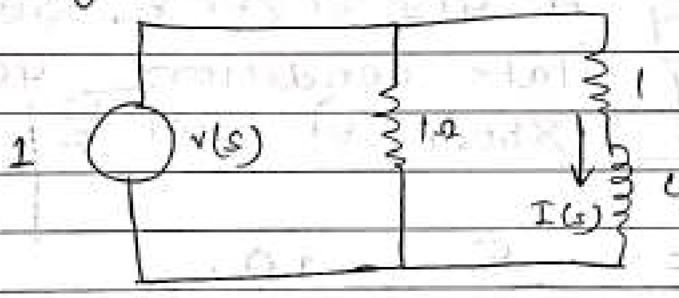
Taking inverse laplace,

$$\boxed{i(t) = 60e^{-2\times10^5 t}} \quad A$$

Q. find i(t) in given figure assume zero initial Response.



Transform in laplace Circuit.



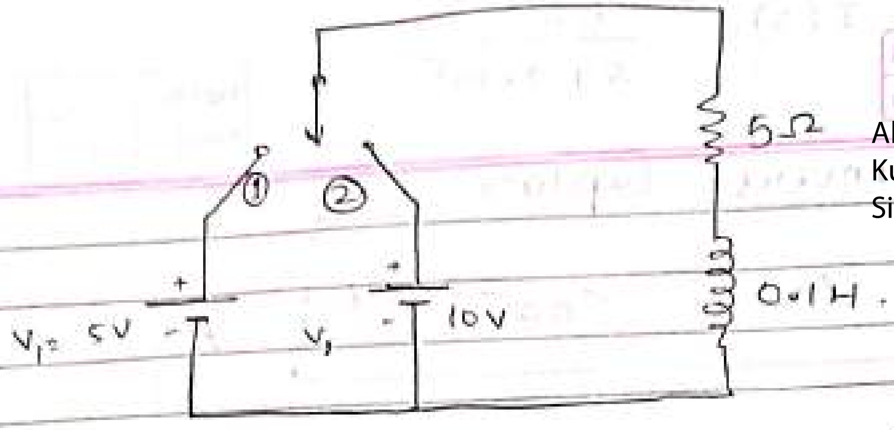
$Z_{eq} = (1+4s)$

$$I(s) = \frac{V(s)}{1+4s} = \frac{1}{1+4s} = \frac{1/4}{s+1/4}$$

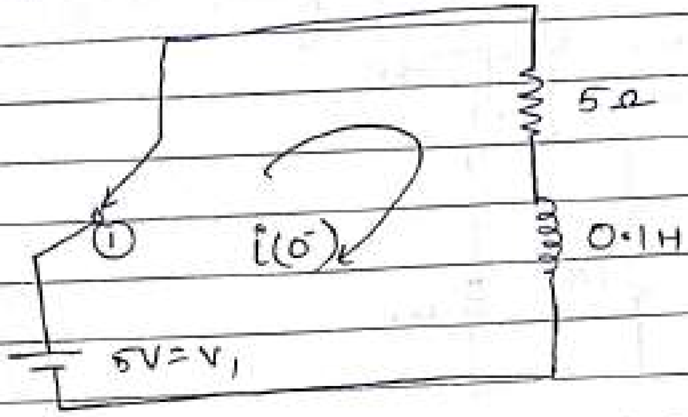
Taking inverse laplace,

$$i(t) = \frac{1}{4}e^{-\frac{1}{4}t}$$

Q In the circuit shown, obtain the expression for the current i(t) when the switch is moved from position (1) to position (2) at t=0.



Switch $t = 0^-$ time तक position ① पर हैं, इसलिए

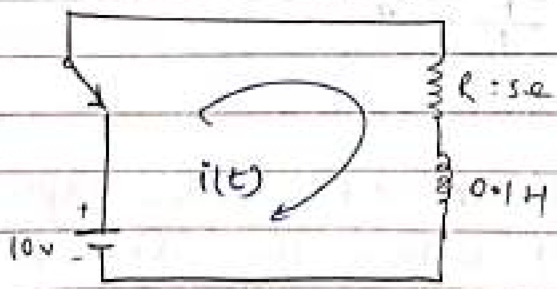


लंबे समय तक ऊपर दिए गए circuit को supply 5V के battery से प्राप्त हो रहा हैं, इस कारण से steady state condition पर Induction short हो जाता हैं।

$$\therefore \; i(0^-) = \frac{5}{5} = 1A.$$

Inductor sudden change को allow नहीं करता इसलिए $i(0^-) = i(0^+) = 1A$.

Now at switch ②,



$$10 = Ri(t) + L\frac{di(t)}{dt}$$

$$10 = 5i(t) + 0.1\frac{di(t)}{dt}$$

$\therefore$ Taking laplace on both side,

$$\frac{10}{S} = 5I(s) + 0.1\left[SI(s) - I(0^+)\right]$$

$$\frac{10}{s}+\frac{0.1}{10} = I(s)\left[5 + 0.1s\right]$$

$$\frac{100+s}{10s} = I(s)\left[5+\frac{s}{10}\right]$$

$$\frac{s+100}{\cancel{10}s} = I(s)\cdot\left(\frac{s+50}{\cancel{10}}\right)$$

$$I(s) = \frac{s+100}{s(s+50)}$$

Inverse Laplace.

$$\therefore \quad i(t) = 2 - e^{-50t} \quad A$$